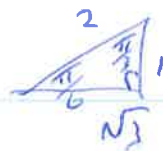
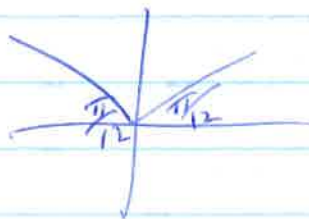


# Lesson 9 - Solutions

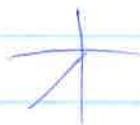


$$\begin{aligned}
 1. \text{ a) } \cos \frac{\pi}{12} &= \cos \left( \frac{3\pi}{12} - \frac{2\pi}{12} \right) \\
 &= \cos \left( \frac{\pi}{4} - \frac{\pi}{6} \right) \\
 &= \cos \frac{\pi}{4} \cos \left( \frac{\pi}{6} \right) + \sin \frac{\pi}{4} \sin \frac{\pi}{6} \\
 &= \frac{\sqrt{2}}{2} \left( \frac{\sqrt{3}}{2} \right) + \frac{\sqrt{2}}{2} \left( \frac{1}{2} \right) \\
 &= \frac{\sqrt{6} + \sqrt{2}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \cos \frac{11\pi}{12} &= -\cos \frac{\pi}{12}
 \end{aligned}$$



$$= \frac{-\sqrt{6} - \sqrt{2}}{4}$$



$$\text{c) } \sin \frac{13\pi}{12}$$

$$= \sin \left( \frac{15\pi}{12} - \frac{2\pi}{12} \right)$$

$$= \sin \left( \frac{5\pi}{4} - \frac{\pi}{6} \right)$$

$$= \sin \frac{5\pi}{4} \cos \frac{\pi}{6} - \cos \frac{5\pi}{4} \sin \frac{\pi}{6}$$

$$\rightarrow = -\frac{\sqrt{2}}{2} \left( \frac{\sqrt{3}}{2} \right) - \left( -\frac{\sqrt{2}}{2} \right) \left( \frac{1}{2} \right)$$

$$= \frac{-\sqrt{6} + \sqrt{2}}{4}$$

Cor use fact that

$$\sin \frac{13\pi}{12} = -\sin \frac{\pi}{12}$$

$$d) \tan \frac{5\pi}{12}$$

$$= \frac{\sin \frac{5\pi}{12}}{\cos \frac{5\pi}{12}}$$

$$= \frac{\sin \left( \frac{3\pi}{12} + \frac{2\pi}{12} \right)}{\cos \left( \frac{3\pi}{12} + \frac{2\pi}{12} \right)}$$

$$= \frac{\sin \left( \frac{\pi}{4} + \frac{\pi}{6} \right)}{\cos \left( \frac{\pi}{4} + \frac{\pi}{6} \right)}$$

$$= \frac{\sin \frac{\pi}{4} \cos \frac{\pi}{6} + \cos \frac{\pi}{4} \sin \frac{\pi}{6}}{\cos \frac{\pi}{4} \cos \frac{\pi}{6} - \sin \frac{\pi}{4} \sin \frac{\pi}{6}}$$

$$= \frac{\frac{\sqrt{2}}{2} \left( \frac{\sqrt{3}}{2} \right) + \frac{\sqrt{2}}{2} \left( \frac{1}{2} \right)}{\frac{\sqrt{2}}{2} \left( \frac{\sqrt{3}}{2} \right) - \frac{\sqrt{2}}{2} \left( \frac{1}{2} \right)}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4} \div \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4} \times \frac{4}{\sqrt{6} - \sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}$$

$$e) \cos 105^\circ$$

$$= \cos(60^\circ + 45^\circ)$$

$$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$= \frac{1}{2} \left( \frac{\sqrt{2}}{2} \right) - \frac{\sqrt{3}}{2} \left( \frac{\sqrt{2}}{2} \right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$2. a) \sin\left(\frac{\pi}{3} + x\right)$$

$$= \sin \frac{\pi}{3} \cos x + \cos \frac{\pi}{3} \sin x$$

$$= \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x$$

$$= \frac{\sqrt{3} \cos x + \sin x}{2}$$

$$b) \sin(2\pi - x)$$

$$= \sin 2\pi \cos x - \cos 2\pi \sin x$$

$$= 0 \cos x - 1 \sin x$$

$$= -\sin x$$

$$c) \tan\left(\frac{\pi}{4} + x\right)$$

$$= \frac{\sin\left(\frac{\pi}{4} + x\right)}{\cos\left(\frac{\pi}{4} + x\right)}$$

$$= \frac{\sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x}{\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x}$$

$$= \frac{\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x}{\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x}$$

$$= \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\left(\div \frac{\sqrt{2}}{2}\right)$$