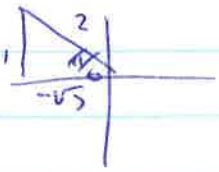
MHF4U

Trig Review - Solutions

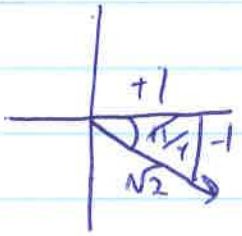
1. a) $\sin\frac{\pi}{6}$
$= \frac{1}{2}$

b) $\csc\frac{\pi}{2}$
$= 1$

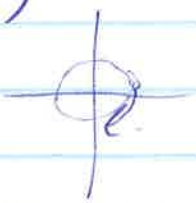
c) $\tan\frac{5\pi}{6}$
$= \frac{-1}{\sqrt{3}}$
$= \frac{-\sqrt{3}}{3}$.



d) $\sec\frac{7\pi}{4}$
$= \sqrt{2}$



e) $\cos\left(\frac{-5\pi}{2}\right)$
$= \cos\left(-2\pi - \frac{\pi}{2}\right)$
$= 0$



2. a) $\frac{4 \text{ revs}}{60 \text{ s}}$
$= \frac{8\pi \text{ rads}}{60 \text{ s}}$
$= \frac{2\pi}{15}$ rads/s.

b) $\frac{2\pi}{15}$ rad/s $\times$ 30s
$= 4\pi$ rads travelled.

$\theta = \frac{a}{r}$

$a = r\theta$

$\Rightarrow a = 3\text{m}\,(4\pi \text{ rads})$
$a = 12\pi \text{ m}.$

3. $\theta = \frac{a}{r}$

$\theta = \frac{22\,cm}{10\,cm}$

$\theta = \frac{11}{5}$ rads.

$A = \pi r^2 \times \frac{11/5}{2\pi}$

$A = \pi (10)^2 \times \frac{11}{10\pi}$

$A = 110\,cm^2$

4.

a) $y = -10 \sin\left[\frac{\pi}{5}(x-1)\right] + 5$

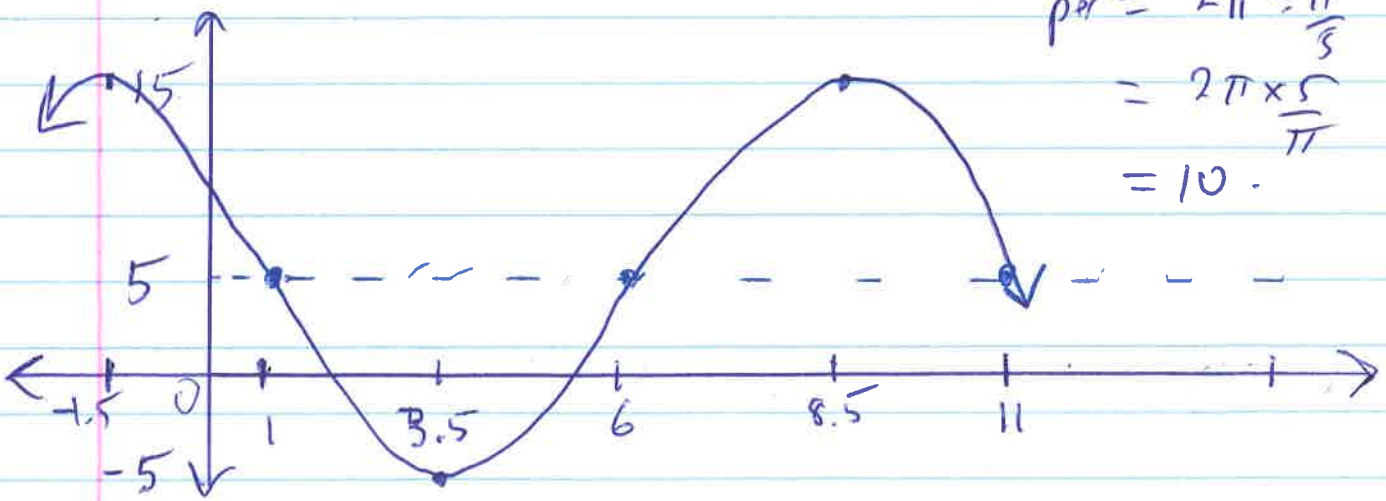
axis at $y = 5$

amp. $= 10$

per $= 2\pi \div \frac{\pi}{5}$

$= 2\pi \times \frac{5}{\pi}$

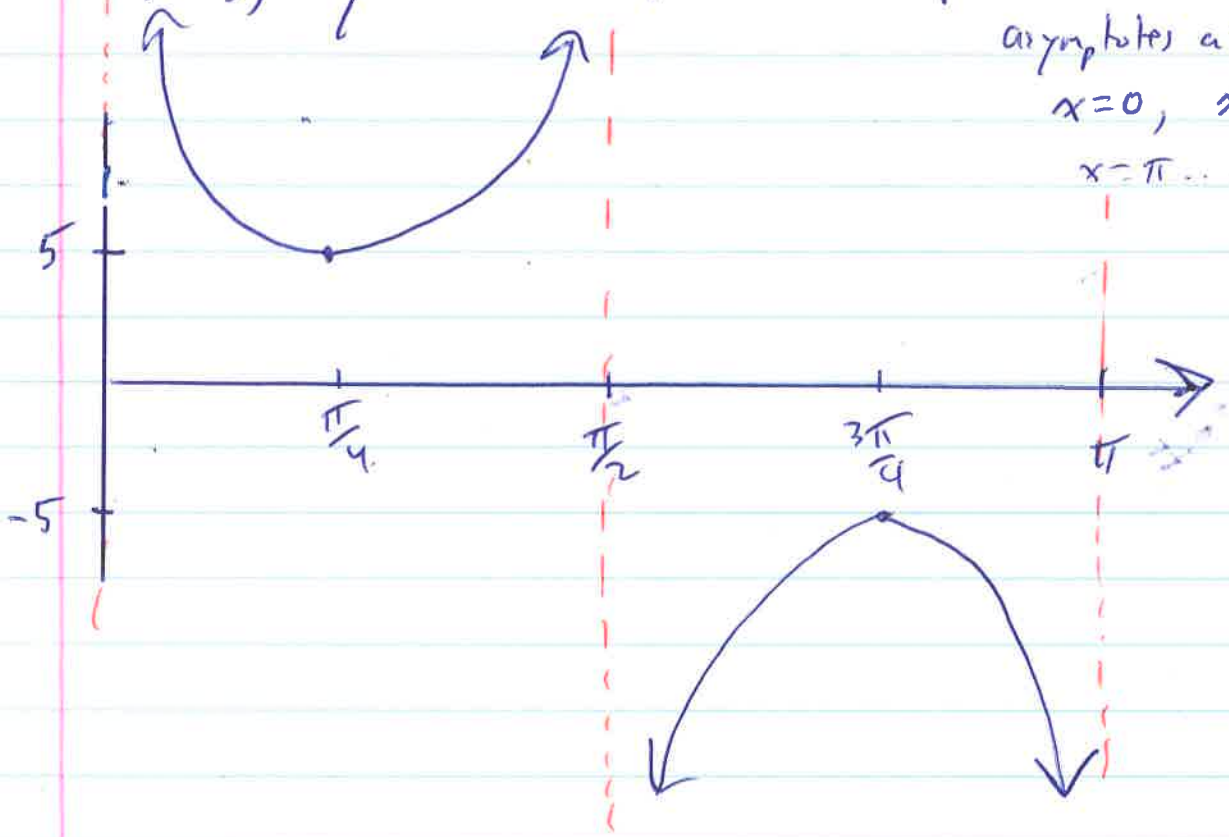
$= 10.$



4 b) $y = 5\csc(2x)$.

period $= \pi$

asymptotes at $x = 0$, $x = \pi/2$, $x = \pi$ ...



5. axis at $y = -2$ amplitude $= 3$.

period $= 6$ so $6 = \frac{2\pi}{k}$

$6k = 2\pi$

$k = \pi/3$

possible equations are: $y = 3\cos[\pi/3(x-1)] - 2$

$y = -3\cos[\pi/3(x+2)] - 2$

$y = 3\sin[\pi/3(x-5.5)] - 2$

(lots of answers).

6. a) $\tan\left(-\frac{\pi}{12}\right)$

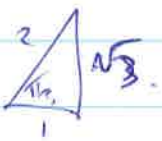
$= -\tan\left(\frac{\pi}{12}\right)$

$= -\tan\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= -\tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= -\left(\frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}\right)$

$= \frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{3}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$



$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$

or $= \frac{1-\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$

$= \frac{1 - 2\sqrt{3} + 3}{-2}$

$= \frac{4 - 2\sqrt{3}}{-2} = \sqrt{3} - 2$

6. b) $\cos \frac{7\pi}{12}$

$= \cos\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2}\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

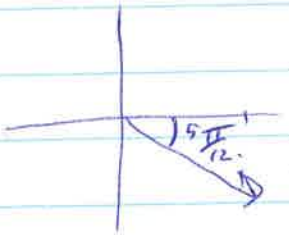
$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}} \quad \times \frac{\sqrt{2}}{\sqrt{2}}$

or

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

c) $\csc \frac{19\pi}{12}$



$= \csc\left(\frac{-5\pi}{12}\right)$

$= -\csc\left(\frac{5\pi}{12}\right)$

$= -\csc\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= -\csc\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \frac{-1}{\sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)}$

$= \frac{-1}{\sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}}$

$= \frac{-1}{\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)}$

$= \frac{-1}{\frac{\sqrt{6}+\sqrt{2}}{4}}$

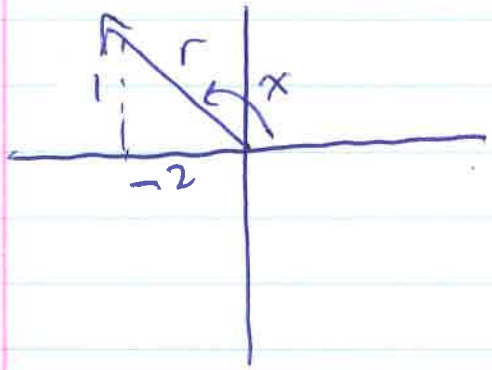
$= \frac{-4}{\sqrt{6}+\sqrt{2}}$

$= \frac{-4(\sqrt{6}-\sqrt{2})}{6-2}$

$= -(\sqrt{6}-\sqrt{2})$

$= \sqrt{2}-\sqrt{6}.$

7. $\cos 2x$ $\quad \tan x = -0.5$



$r^2 = (-1)^2 + 2^2$

$r = \sqrt{5}$

$\sin x = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\cos x = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$

$\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\frac{-2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2$

$= \frac{4}{5} - \frac{1}{5}$

$\boxed{\cos 2x = \frac{3}{5}}$

8. a) $LS = \sin^2 x$ $\qquad RS = \dfrac{1-\cos 2x}{2}$

$$RS = \frac{1-(\cos^2 x - \sin^2 x)}{2}$$

$$RS = \frac{1-\cos^2 x + \sin^2 x}{2}$$

$$RS = \frac{1-(1-\sin^2 x)+\sin^2 x}{2}$$

$$RS = \frac{\sin^2 x + \sin^2 x}{2}$$

$$RS = \frac{2\sin^2 x}{2}$$

$$RS = \sin^2 x$$

$$= LS$$

□

b) $LS = \dfrac{2\sec^2 x - 2\tan^2 x}{\csc x}$

$= \dfrac{\frac{2}{\cos^2 x} - 2\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\sin x}}$

$= \dfrac{2 - 2\sin^2 x}{\cos^2 x} \times \sin x$

$= \dfrac{2\sin x - 2\sin^3 x}{\cos^2 x}$

$= \dfrac{2\sin x - 2\sin^3 x}{(1 - \sin^2 x)}$

$= \dfrac{2\sin x(1 - \sin^2 x)}{(1 - \sin^2 x)}$

$= 2\sin x.$ □

$RS = \sin 2x \sec x$

$= \dfrac{\sin 2x}{\cos x}$

$= \dfrac{2\sin x \cos x}{\cos x}$

$= 2\sin x.$

8. c) $RS = \dfrac{\sin 2x}{1-\cos 2x}$

$= \dfrac{2\sin x \cos x}{1-(\cos^2 x - \sin^2 x)}$

$= \dfrac{2\sin x \cos x}{1-\cos^2 x + \sin^2 x}$

$= \dfrac{2\sin x \cos x}{1-(1-\sin^2 x)+\sin^2 x}$

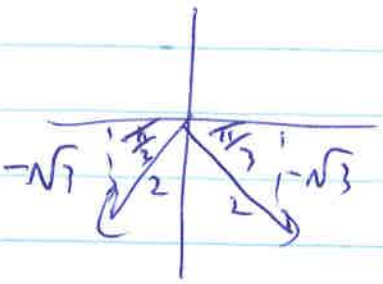
$= \dfrac{2\sin x \cos x}{2\sin^2 x}$

$= \dfrac{\cos x}{\sin x}$

$= \cot x.$

□.

9. a) $\sin x = -\frac{\sqrt{3}}{2}$



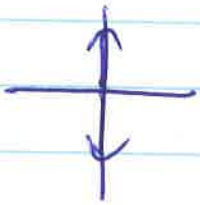
$x = \pi + \frac{\pi}{3} + 2\pi k$

$\boxed{x = \frac{4\pi}{3} + 2\pi k,\ k \in I}$

or $x = 2\pi - \frac{\pi}{3} + 2\pi k$

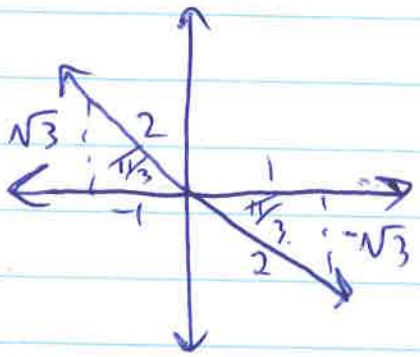
$\boxed{x = \frac{5\pi}{3} + 2\pi k,\ k \in I}$

b) $\cos x = 0$



$x = \frac{\pi}{2} + \pi k,\ k \in I.$

c) $\tan 2x = -\sqrt{3}$



$2x = \frac{2\pi}{3} + \pi k \quad k \in I$

$\boxed{x = \frac{\pi}{3} + \frac{\pi}{2} k,\ k \in I}$

Q: d) $\cos 2x + \sin x = 1$

$\cos^2 x - \sin^2 x + \sin x = 1$

$(1 - \sin^2 x) - \sin^2 x + \sin x = 1$

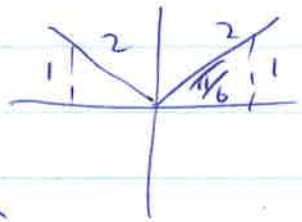
$-2\sin^2 x + 1 + \sin x = 1$

$-2\sin^2 x + \sin x = 0$

$\sin x(-2\sin x + 1) = 0$

$\sin x = 0$ or $-2\sin x + 1 = 0$

$\sin x = \frac{1}{2}$



$x = \pi k, k \in I$

or $x = \frac{\pi}{6} + 2\pi k, k \in I$

or $x = \frac{5\pi}{6} + 2\pi k, k \in I$

e) $2\cos^2 x = \sin x - 1$

$2\cos^2 x - \sin x + 1 = 0$

$2(1-\sin^2 x) - \sin x + 1 = 0$

$-2\sin^2 x + 2 - \sin x + 1 = 0$

$-2\sin^2 x - \sin x + 3 = 0$

$2\sin^2 x + \sin x - 3 = 0$

$(2\sin x + 3)(\sin x - 1) = 0$

$2\sin x + 3 = 0$ or $\sin x = 1$

$\sin x = -\frac{3}{2}$

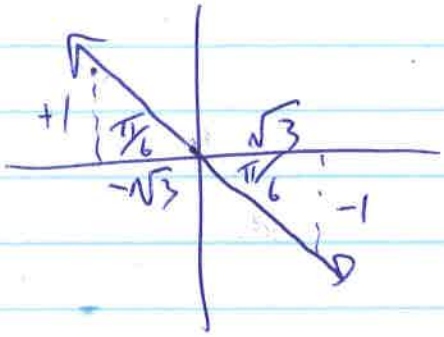
(not possible)

$x = \frac{\pi}{2} + 2\pi k, \; k \in I$

f) $\frac{\cos x}{\sqrt{3}} = -\sin x$

$\frac{-1}{\sqrt{3}} = \frac{\sin x}{\cos x}$

$\tan x = \frac{-1}{\sqrt{3}}$



$\boxed{x = \frac{5\pi}{6} + \pi k,\ k \in I}$

10. Solution is $f(x) = g(x)$

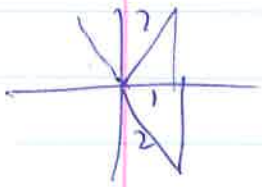
$$\cos 2x = -\cos x$$

$$2\cos^2 x - 1 = -\cos x$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$\cos 2x$
$= \cos^2 x - \sin^2 x$
$= \cos^2 x - (1 - \cos^2 x)$
$= 2\cos^2 x - 1$



$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos x = -1$

$x = \pi$

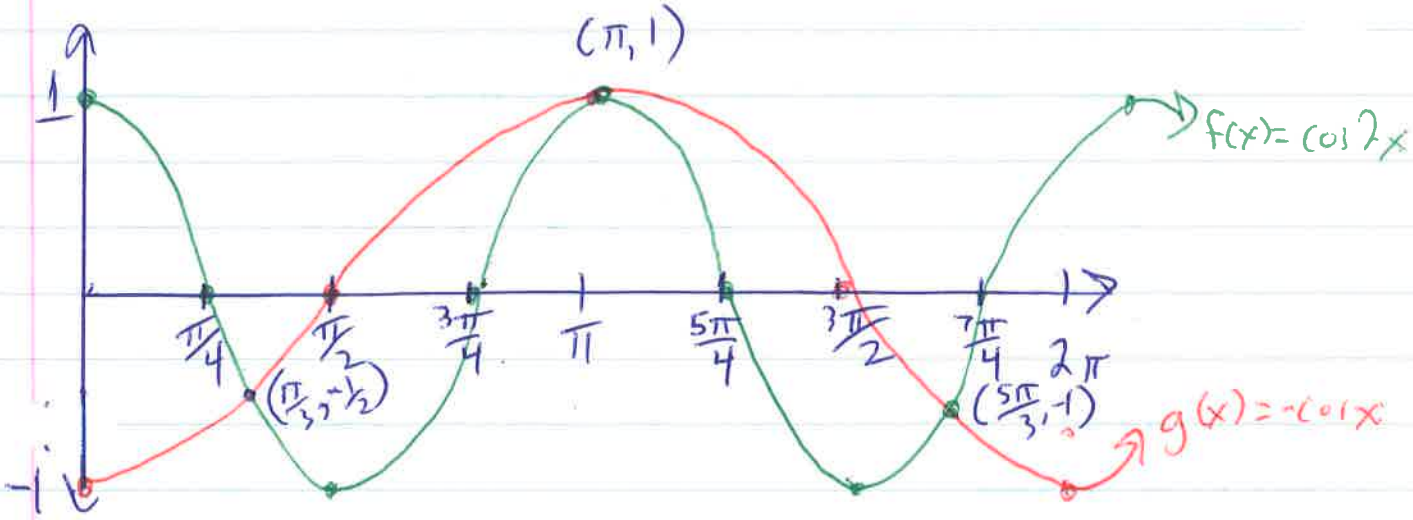
$g(\pi) = -\cos \pi$
$= \underline{1}$

$g(\frac{\pi}{3}) = -\cos \frac{\pi}{3}$
$= -\frac{1}{2}$

$g(\frac{5\pi}{3}) = -\cos \frac{5\pi}{3}$
$= -\frac{1}{2}$

∴ points of intersection are

$(\frac{\pi}{3}, -\frac{1}{2})$ $(\pi, 1)$ and $(\frac{5\pi}{3}, -\frac{1}{2})$.


(π, 1)
1
-1
π/4
π/2
(π/3, -1/2)
3π/4
π
5π/4
3π/2
7π/4
2π
(5π/3, -1)
f(x) = cos 2x
g(x) = -cos x