

Trigonometry Quiz #2

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MHF4U

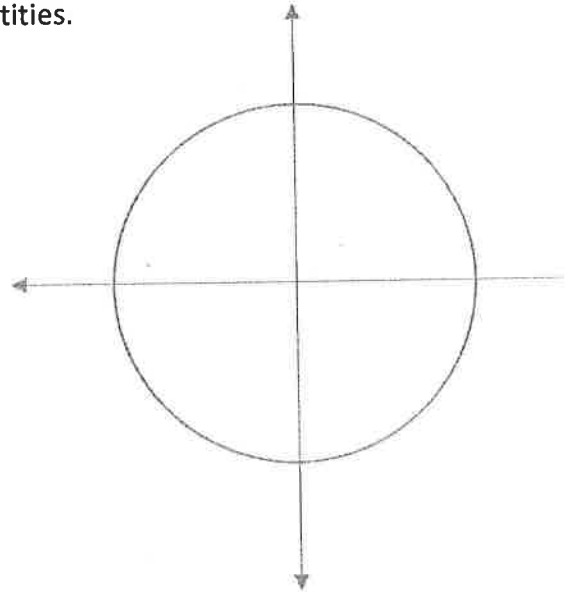
Name: Solutions

****no calculators allowed****

1. Prove that $\cos(A - B) = \cos A \cos B + \sin A \sin B$. Use the unit circle in your proof, and do not use any of the other addition/subtraction identities.

[10 marks]

see notes!



2. Prove that $\sin 2x = 2 \sin x \cos x$, using any of the addition/subtraction identities.

[3 marks]

$$\begin{aligned} \sin 2x &= \sin(x+x) \\ &= \sin x \cos x + \cos x \sin x \quad (\text{sine addition identity}) \\ &= 2 \sin x \cos x \end{aligned}$$

3. Find an exact value for $\sin \frac{11\pi}{12}$.

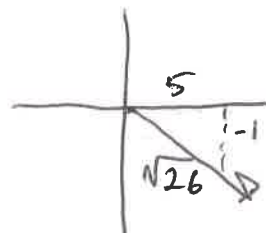
[4 marks]

$$\begin{aligned} \sin\left(\frac{11\pi}{12}\right) &= \sin\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right) \\ &= \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) \\ &= \sin \frac{2\pi}{3} \cos \frac{\pi}{4} + \cos \frac{2\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right) \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

4. Suppose that $\tan x = \frac{-1}{5}$, and $\pi < x < 2\pi$.

[7 marks]

- Find an exact value for $\sin 2x$
- Find an exact value for $\cos 2x$.
- Find an exact value for $\sin(2x + \frac{\pi}{4})$

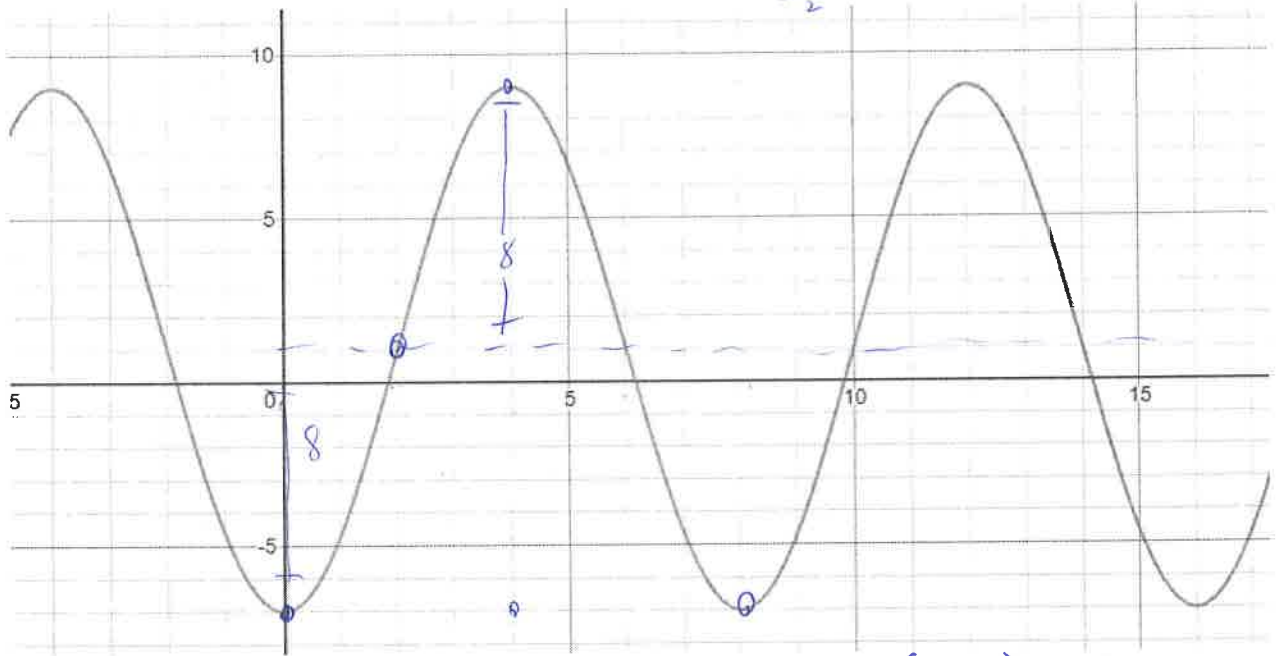


$$\begin{aligned} \text{a) } \sin 2x &= 2 \sin x \cos x \\ &= 2 \left(\frac{-1}{\sqrt{26}}\right) \left(\frac{5}{\sqrt{26}}\right) \\ &= \frac{-10}{26} = \frac{-5}{13} \end{aligned}$$

$$\begin{aligned} \text{b) } \cos 2x &= \cos^2 x - \sin^2 x \\ &= \left(\frac{5}{\sqrt{26}}\right)^2 - \left(\frac{-1}{\sqrt{26}}\right)^2 \\ &= \frac{25}{26} - \frac{1}{26} \\ &= \frac{24}{26} = \frac{12}{13} \end{aligned}$$

$$\begin{aligned} \text{c) } \sin\left(2x + \frac{\pi}{4}\right) &= \sin 2x \cos \frac{\pi}{4} + \cos 2x \sin \frac{\pi}{4} \\ &= \frac{-5}{13} \frac{\sqrt{2}}{2} + \frac{12}{13} \frac{\sqrt{2}}{2} \\ &= \frac{-5\sqrt{2} + 12\sqrt{2}}{26} \\ &= \frac{7\sqrt{2}}{26} \end{aligned}$$

5. Find two equations for the sinusoidal graph below. One of your equations should use sine and the other cosine. [5 marks]



per. = 8

$$s = \frac{2\pi}{k}$$

$$8k = 2\pi$$

$$k = \frac{\pi}{4}$$

$$y = -8\cos\left(\frac{\pi}{4}x\right) + 1$$

$$y = 8\sin\left[\frac{\pi}{4}(x-2)\right] + 1$$

6. Sketch the graph of $y = 5 \sin\left[\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right] + 10$ over the interval $0 \leq x \leq 2\pi$. [5 marks]

