MCR 3U Final Exam Review

Handout # 1. ① $f(x) = 3x^2 - 2x + 1$ ② $g(x) = 2x - 3$

a) $f(2) = 3(2)^2 - 2(2) + 1$
$= 12 - 4 + 1$
$= 9$

b) $f(2a) = 3(2a)^2 - 2(2a) + 1$
$= 3(4a^2) - 4a + 1$
$= 12a^2 - 4a + 1$

c) $f(4) - 2g(-1)$
$= 3(4)^2 - 2(4) + 1 - 2[2(-1) - 3]$
$= 3(16) - 8 + 1 - 2(-5)$
$= 48 - 8 + 1 + 10$
$= 51$

d) set ① = ②
$3x^2 - 2x + 1 = 2x - 3$
$3x^2 - 4x + 4 = 0$
$D = b^2 - 4ac$
$= (-4)^2 - 4(3)(4)$
$= 16 - 48$
$= -32$
$\because D < 0$
$\therefore$ There are no sol'ns/POI's

e) for $g^{-1}(x)$
$x = 2y - 3$ ← x and y switch positions for the inverse equ'n
$x + 3 = 2y$
$y = \frac{x+3}{2}$
$\therefore g^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$

f) If 6 is the x-coord. for the inverse function, then 6 is the f(x) or y value for the original function.

set $f(x) = 6$
$6 = 3x^2 - 2x + 1$
$0 = 3x^2 - 2x - 5$
$0 = (3x - 5)(x + 1)$
$x = \frac{5}{3}$ or -1
$\therefore f^{-1}(6) = \frac{5}{3}$ or -1

$\boxed{\text{OR}}$ But who wants to do all of that?

f) (i) for $f^{-1}(x)$
$x = 3y^2 - 2y + 1$
$x = 3\left[\left(y^2 - \frac{2}{3}y + \frac{1}{9}\right) - \frac{1}{9}\right] + 1$ ← Use CTS
$x = 3\left(y - \frac{1}{3}\right)^2 - \frac{1}{3} + 1$
$x = 3\left(y - \frac{1}{3}\right)^2 + \frac{2}{3}$ ← Use SAMDEB next
$x - \frac{2}{3} = 3\left(y - \frac{1}{3}\right)^2$
$\frac{x - \frac{2}{3}}{3} = \left(y - \frac{1}{3}\right)^2$
$\pm\sqrt{\frac{x}{3} - \frac{2}{9}} = y - \frac{1}{3}$
$\therefore f^{-1}(x) = \pm\sqrt{\frac{x}{3} - \frac{2}{9}} + \frac{1}{3}$

(ii) $f^{-1}(6) = \pm\sqrt{2 - \frac{2}{9}} + \frac{1}{3}$
$= \pm\sqrt{\frac{16}{9}} + \frac{1}{3}$
$= \pm\frac{4}{3} + \frac{1}{3}$
$= \frac{5}{3}$ or -1

Final Exam Review

Handout Q #2

$f(x)=\sqrt{x}$ $\quad g(x) = 2\sqrt{-(x+1)} - 3$

a) $g(x) = 2f[-(x+1)]-3$

b) $(x,y) \rightarrow (-x-1,\ 2y-3)$

$(0,0) \rightarrow (-1,-3)$
$(1,1) \rightarrow (-2,-1)$
$(4,2) \rightarrow (-5,1)$
$(9,3) \rightarrow (-10,3)$

These would be key points for the radical function $g(x)$.

c) D: $\{x \in R \mid x \leq -1\}$

R: $\{g(x) \in R \mid g(x) \geq -3\}$

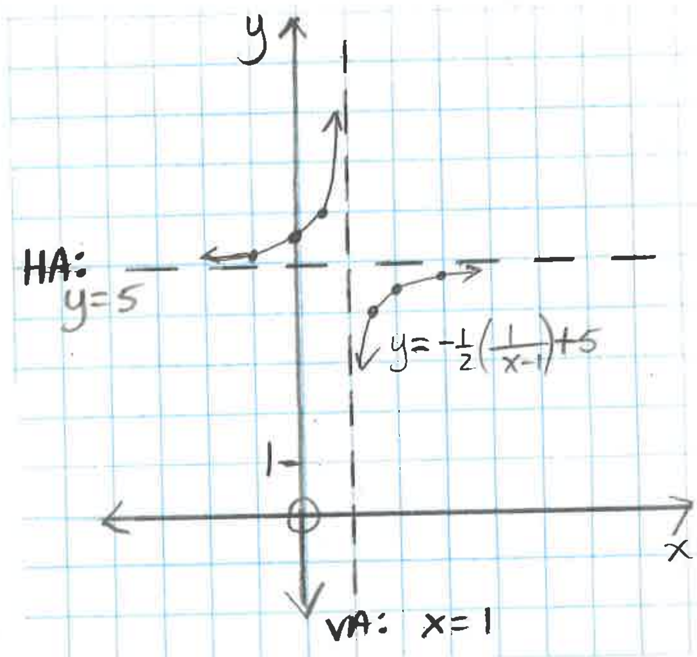
Handout Q #3

$y=\frac{1}{x} \rightarrow y = \frac{-1}{2(x-1)} + 5$ or $y = -\frac{1}{2}\left(\frac{1}{x-1}\right)+5$

$(x,y) \rightarrow (x+1,\ -\frac{1}{2}y+5)$

$(-2,-\frac{1}{2}) \rightarrow (-1, 5\frac{1}{4})$
$(-1,-1) \rightarrow (0, 5\frac{1}{2})$
$(-\frac{1}{2},-2) \rightarrow (\frac{1}{2}, 6)$
$(\frac{1}{2}, 2) \rightarrow (\frac{3}{2}, 4)$
$(1,1) \rightarrow (2, 4\frac{1}{2})$
$(2,\frac{1}{2}) \rightarrow (3, 4\frac{3}{4})$

VA: $x=0 \rightarrow$ VA: $x=1$
HA: $y=0 \rightarrow$ HA: $y=5$



Handout Q #4

a) $4x^2-12x+9$ PST
$= (2x-3)^2$

b) $2x^2-6x+4$ CF!!
$= 2(x^2-3x+2)$ GT
$= 2(x-2)(x-1)$

c) $x^2+4x+4-y^2$ Factor PST
$= (x+2)^2 - y^2$ DS
$= (x+2+y)(x+2-y)$
$= (x+y+2)(x-y+2)$

d) x^4-1 DS
$= (x^2+1)(x^2-1)$ ← DS
$= (x^2+1)(x+1)(x-1)$

e) $4x^3-8x^2-x+2$ Factor by grouping
$= 4x^2(x-2)-(x-2)$
$= (4x^2-1)(x-2)$ DS!
$= (2x+1)(2x-1)(x-2)$

f) $9(x-2)^2-4x^2$ DS!
$= [3(x-2)+2x][3(x-2)-2x]$
$= (3x-6+2x)(3x-6-2x)$
$= (5x-6)(x-6)$

Handout Q #5

a) $-2\sqrt{8}$

$= -2\sqrt{4\times 2}$

$= -4\sqrt{2}$

b) $\sqrt{24}$

$= 2\sqrt{6}$

c) $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{3}$

d) $\frac{5\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{5\sqrt{6}}{3}$

e) $(5-\sqrt{2})(4+3\sqrt{2})$

$= 20 + 15\sqrt{2} - 4\sqrt{2} - 3(2)$

$= 14 + 11\sqrt{2}$

f) $\frac{2}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$ Mult. numerator and denom by the conjugate of $3+\sqrt{2}$ (which is $3-\sqrt{2}$)

$= \frac{6-2\sqrt{2}}{9-2}$

$= \frac{6-2\sqrt{2}}{7}$

Handout Q #6 $\boxed{\text{Text}}$ p.134 5, 6, 7

5. a) $3m(m-1) + 2m(1-m)$ ← Factor −1 out

$= 3m(m-1) - 2m(m-1)$ ← Factor by grouping

$= (3m-2m)(m-1)$

$= m(m-1)$

b) $x^2 - 27x + 72$ GT

$= (x-3)(x-24)$

c) $15x^2 - 7xy - 2y^2$ ← GT. Chart or decomposition.

1	3	5	15	1
15	5	3	1	2

Diff of 7

$= (5x+y)(3x-2y)$

d) $(2x-y+1)^2 - (x-y-2)^2$ DS!

$= [2x-y+1-(x-y-2)][2x-y+1+(x-y-2)]$

$= (x+3)(3x-2y-1)$

e) $5xy - 10x - 3y + 6$ Factor by grouping

$= 5x(y-2) - 3(y-2)$

$= (5x-3)(y-2)$

f) $p^2 - m^2 + 6m - 9$ ← Create PST by factoring −1 out of last 3 terms

$= p^2 - (m^2 - 6m + 9)$

$= p^2 - (m-3)^2$ ← Factored PST. Creates a DS!

$= [p-(m-3)][p+(m-3)]$

$= (p-m+3)(p+m-3)$

$= (-m+p+3)(m+p-3)$ ← Ideally terms are in alphabetical order w/ +ve leading coefficient.

$= -(m-p-3)(m+p-3)$

6. x-int, y=0

$0 = x^3 - 4x^2 - x + 4$

$0 = x^2(x-4) - (x-4)$

$0 = (x^2-1)(x-4)$

$0 = (x+1)(x-1)(x-4)$

∴ The x-int. are at −1, 1, and 4.

7. a) $\frac{4a^2b}{5ab^3} \div \frac{6a^2\cancel{b}}{35a\cancel{b}}$

$= \frac{4a}{5b^2} \div \frac{6a}{35}$

$= \frac{{}^2\cancel{4}\cancel{a}}{\cancel{5}_1 b^2} \cdot \frac{\cancel{35}^7}{\cancel{6}_3\cancel{a}}$

$= \frac{14}{3b^2}$

$a \neq 0,\ b \neq 0$

b) $\frac{x-2}{x^2-x-12} \times \frac{2x-8}{x^2-4x+4}$

$= \frac{\cancel{(x-2)}}{\cancel{(x-4)}(x+3)} \times \frac{2\cancel{(x-4)}}{(x-2)^{\cancel{2}}}$

$= \frac{2}{(x+3)(x-2)}$

$x \neq -3, 2, 4$

c) $\frac{5}{t^2-7t-18} + \frac{6}{t+2}$

$= \frac{5}{(t-9)(t+2)} + \frac{6}{(t+2)}$

$= \frac{5+6(t-9)}{(t-9)(t+2)}$

$= \frac{6t-49}{(t-9)(t+2)}$

$t \neq -2, 9$

d) $\frac{4x}{6x^2+13x+6} - \frac{3x}{4x^2-9}$

$= \frac{4x}{(3x+2)(2x+3)} - \frac{3x}{(2x+3)(2x-3)}$

$= \frac{4x(2x-3) - 3x(3x+2)}{(3x+2)(2x+3)(2x-3)}$

$= \frac{8x^2-12x-9x^2-6x}{(3x+2)(2x+3)(2x-3)}$

$= \frac{-x^2-18x}{(3x+2)(2x+3)(2x-3)}$

$x \neq -\frac{3}{2}, -\frac{2}{3}, \frac{3}{2}$

RW

1 2 3 6	1 2
6 3 2 1	6 3

sum of 13

Handout Q # 7 $\boxed{\text{Text}}$ p. 133 # 15

#15. a) $\dfrac{1}{2x} - \dfrac{7}{3x^2} + \dfrac{4}{x^3}$

$$= \frac{3x^2 - 14x + 4(6)}{6x^3}$$

$$= \frac{3x^2 - 14x + 24}{6x^3}$$

$x \neq 0$

b) $\dfrac{3x}{x+2} + \dfrac{4x}{x-6}$

$$= \frac{3x(x-6) + 4x(x+2)}{(x+2)(x-6)}$$

$$= \frac{3x^2 - 18x + 4x^2 + 8x}{(x+2)(x-6)}$$

$$= \frac{7x^2 - 10x}{(x+2)(x-6)}$$

$x \neq -2, 6$

c) $\dfrac{6x}{x^2-5x+6} - \dfrac{3x}{x^2+x-12}$

$$= \frac{6x}{(x-3)(x-2)} - \frac{3x}{(x+4)(x-3)}$$

$$= \frac{6x(x+4) - 3x(x-2)}{(x-3)(x-2)(x+4)}$$

$$= \frac{6x^2 + 24x - 3x^2 + 6x}{(x-3)(x-2)(x+4)}$$

$$= \frac{3x^2 + 30x}{(x-3)(x-2)(x+4)}$$

$x \neq -4, 2, 3$

d) $\dfrac{2(x-2)^2}{x^2+6x+5} \times \dfrac{3x+15}{(2-x)^2}$

$$= \frac{2(x-2)^2}{\cancel{(x+5)}(x+1)} \times \frac{3\cancel{(x+5)}}{(2-x)(2-x)}$$

$$= \frac{2(x-2)^2}{(x+1)} \times \frac{3}{(-1)(-1)(x-2)(x-2)}$$

$$= \frac{2\cancel{(x-2)^2}}{(x+1)} \times \frac{3}{\cancel{(x-2)^2}}$$

$$= \frac{6}{x+1}$$

$x \neq -5, -1, 2$

e) $\dfrac{(x-2y)^2}{x^2-y^2} \div \dfrac{(x-2y)(x+3y)}{(x+y)^2}$

$$= \frac{(x-2y)^{\cancel{2}}}{\cancel{(x+y)}(x-y)} \times \frac{(x+y)^{\cancel{2}}}{\cancel{(x-2y)}(x+3y)}$$

$$= \frac{(x-2y)(x+y)}{(x-y)(x+3y)}$$

$x \neq \pm y, 2y, -3y$

f) $\dfrac{2b-5}{b^2-2b-15} + \dfrac{3b}{b^2+b-30} \times \dfrac{b^2+8b+12}{b+3}$

$$= \frac{2b-5}{(b-5)(b+3)} + \frac{3b}{\cancel{(b+6)}(b-5)} \times \frac{\cancel{(b+6)}(b+2)}{(b+3)}$$

$$= \frac{2b-5}{(b-5)(b+3)} + \frac{3b(b+2)}{(b-5)(b+3)}$$

$$= \frac{2b-5+3b^2+6b}{(b-5)(b+3)}$$

$$= \frac{3b^2+8b-5}{(b-5)(b+3)}$$

$b \neq -6, -3, 5$

Handout Q #8 $\boxed{\text{Text}}$ p. 202 5, 12, 13, 18

#5. $h(t) = -4.9t^2 + 28t + 2$

$= -4.9\left[(t^2 - 5.71t + 8.15) - 8.15\right] + 2$

$= -4.9(t - 2.855)^2 + 39.935 + 2$

$= -4.9(t - 2.855)^2 + 41.935$

∴ The football reaches an approx max height of 42 m after about 2.9 s.

#12. x-int, $f(x) = 0$

$0 = 2x^2 + x - 15$

$0 = (2x - 5)(x + 3)$

∴ The x-int are at $\left(\frac{5}{2}, 0\right)$ and $(-3, 0)$.

RW →

$t = \dfrac{-200 \pm \sqrt{40000 + 780{,}000}}{6}$

$= \dfrac{-200 \pm \sqrt{820\,000}}{6}$

$= \dfrac{\cancel{-200}^{-100} \pm \cancel{2}\sqrt{205\,000}}{\cancel{6}_3}$

$= \dfrac{-100 \pm \sqrt{205\,000}}{3}$

#13 a) $P(13) = 12(13)^2 + 800(13) + 40\,000$

$= 2028 + 10\,400 + 40\,000$

$= 52\,428$

∴ According the model, the pop^{n} will grow to 52 428 people by 2020.

b) set $P(t) = 300\,000$

$300\,000 = 12t^2 + 800t + 40000$

$0 = 12t^2 + 800t - 260\,000$

$0 = 4(3t^2 + 200t - 65000)$

$t = \dfrac{-100 \pm \sqrt{205\,000}}{3}$

$\doteq 117.58$ or $\cancel{-184.26}$ inadmissible

∴ The pop^{n} is predicted to reach 300,000 117.6 yrs after the yr 2007 which is a little more than half-way through the yr 2124.

#18. (i) $x = 2 \pm \sqrt{3}$

$x - 2 = \pm\sqrt{3}$

$(x-2)^2 = (\pm\sqrt{3})^2$

$x^2 - 4x + 4 = 3$

$x^2 - 4x + 1 = 0$

∴ The general equ^{n} is $f(x) = a(x^2 - 4x + 1)$

(ii) sub $(2, 5)$

$5 = a\left[(2)^2 - 4(2) + 1\right]$

$5 = -3a$

$\boxed{a = -\frac{5}{3}}$

∴ The equ^{n} is $f(x) = -\frac{5}{3}(x^2 - 4x + 1)$

or $f(x) = -\frac{5}{3}x^2 + \frac{20}{3}x - \frac{5}{3}$

Handout Q#9 $\boxed{\text{Text}}$ p. 204 6 a, b 7, 8, 9

#6. a) $(2-\sqrt{8})(3+\sqrt{2})$
$= 6+2\sqrt{2}-3\sqrt{8}-\sqrt{16}$
$= 6+2\sqrt{2}-6\sqrt{2}-4$
$= 2-4\sqrt{2}$

6. b) $(3+\sqrt{5})(5-\sqrt{10})$
$= 15-3\sqrt{10}+5\sqrt{5}-\sqrt{50}$
$= 15-3\sqrt{10}+5\sqrt{5}-5\sqrt{2}$

7. $\because$ there is only 1 root
$\therefore D=0$
$D=b^2-4ac$
$0=(-4)^2-4(k)(k)$
$0=16-4k^2$
$4k^2=16$
$k^2=4$
$\boxed{k=\pm 2}$

8. ① $g(x)=6x-5$ ② $f(x)=2x^2-3x+2$
set ① = ② for POI's
(i) $6x-5=2x^2-3x+2$
$0=2x^2-9x+7$
$D=b^2-4ac$
$=81-4(2)(7)$
$=81-56$
$=25$
$\because D>0$
$\therefore x=\dfrac{-b\pm\sqrt{D}}{2a}$
$=\dfrac{9\pm\sqrt{25}}{4}$
$=\dfrac{9\pm5}{4}$
$=\dfrac{7}{2}$ or 1

(ii) $g(1)=6(1)-5$
$=1$
and $g\left(\frac{7}{2}\right)=\cancel{6}^{3}\left(\frac{7}{\cancel{2}_{1}}\right)-5$
$=21-5$
$=16$
$\therefore$ The POI's are $(1,1)$ and $\left(\frac{7}{2},16\right)$.

9. Vertex: $\overset{h}{(4},\overset{k}{3)}$
$a=-1$
$y=a(x-h)^2+k$
$=-1(x-4)^2+3$
$=-(x^2-8x+16)+3$
$=-x^2+8x-13$
$\therefore$ The equ^{n} in standard form is $y=-x^2+8x-13$.

Handout Q#10

a) $-(-3125)^{-2/5}$
$=\dfrac{-1}{(-3125)^{2/5}}$
$=\dfrac{-1}{(-5)^2}$
$=\dfrac{-1}{25}$

b) $\dfrac{(8^{-1}+2^{-3})16^{0}}{(4^{-1}+2^{-2})}$
$=\left(\frac{1}{8}+\frac{1}{2^3}\right)(1)\div\left(\frac{1}{4}+\frac{1}{2^2}\right)$
$=\left(\frac{1}{8}+\frac{1}{8}\right)\div\left(\frac{1}{4}+\frac{1}{4}\right)$
$=\frac{2}{8}\div\frac{2}{4}$
$=\frac{1}{4}\div\frac{1}{2}$
$=\frac{1}{\cancel{4}_2}\times\frac{\cancel{2}^1}{1}$
$=\frac{1}{2}$

c) $\dfrac{5^{4/3}}{5^{-1/3}}$
$=5^{5/3}$
$=\left(\sqrt[3]{5}\right)^5$

Handout #11 $\boxed{\text{Text}}$ p.270 3,4 p.268 11 a,d, 12

p.270 #3 a) $(-3x^2y)^3(-3x^{-3}y)^2$

$= -27x^6y^3(9x^{-6}y^2)$

$= -243y^5$

b) $\dfrac{(5a^{-1}b^2)^{-2}}{125a^5b^{-3}}$

$= \dfrac{(5)^{-2}a^2b^{-4}}{125a^5b^{-3}}$

$= \dfrac{1}{3125a^3b}$

3.c) $\sqrt[5]{\dfrac{1024(x^{-1})^{10}}{(2x^{-3})^5}}$

$= \left(\dfrac{1024x^{-10}}{2^5x^{-15}}\right)^{\frac{1}{5}}$

$= \dfrac{\cancel{4}^{2}x^{-2}}{\cancel{2}_{1}x^{-3}}$

$= 2x$

d) $\dfrac{(8x^6y^{-3})^{1/3}}{(2xy)^3}$

$= \dfrac{\cancel{2}^{1}x^2y^{-1}}{\cancel{8}_{4}x^3y^3}$

$= \frac{1}{4}x^{-1}y^{-4}$

$= \dfrac{1}{4xy^4}$

4. a) $I = 100(0.964)^n$ where 'I' is the intensity of light, 'n' is the number of gels used

b) sub $n=3$

$I = 100(0.964)^3$

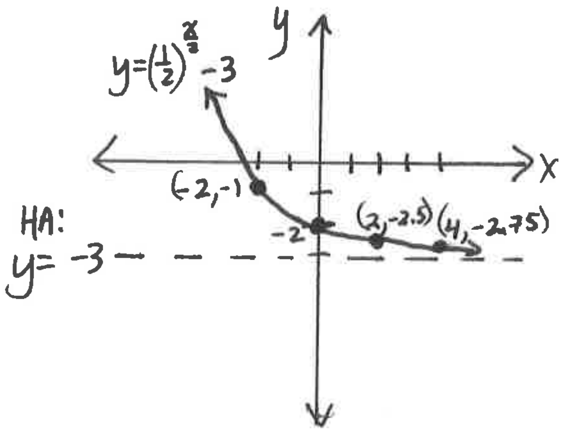
$\doteq 89.6\%$

∴ When 3 gels are used the percentage of light left is approx. 89.6%

c) This is an example of exponential decay because the intensity of light is decreasing exponentially as the number of gels increases. It is evident from the decay factor of 0.964 i.e. with each gel applied the intensity is only 96.4% what it was previously.

Handout #11 (still) $\boxed{\text{Text}}$ p. 268 11 a, d 12

11. a) $y = \left(\frac{1}{2}\right)^{\frac{x}{2}} - 3$



The base function is $y = \left(\frac{1}{2}\right)^x$. It is being horizontally expanded by a factor of 2 and translated down 3 units.

$(x, y) \rightarrow (2x, y-3)$

$(-1, 2) \rightarrow (-2, -1)$

$(0, 1) \rightarrow (0, -2)$

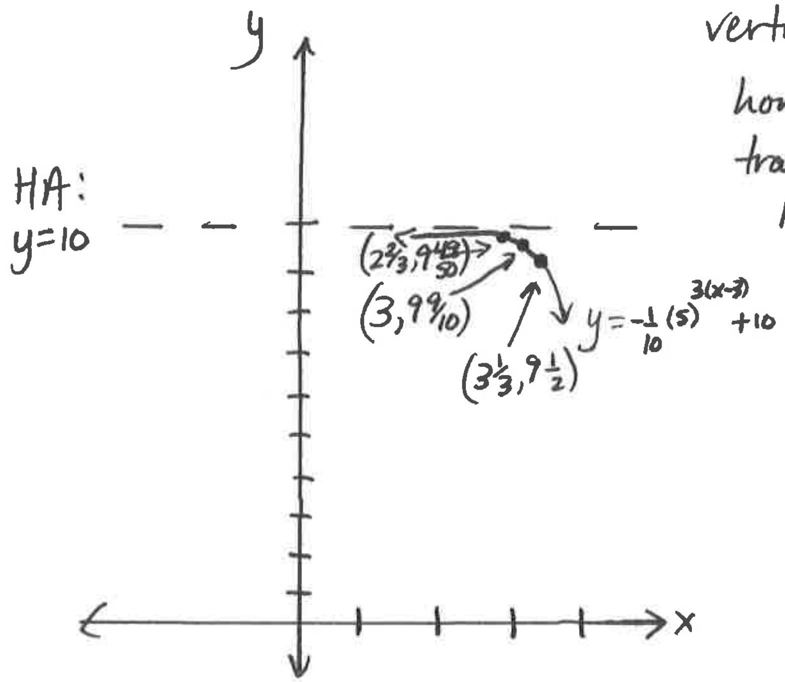
$(1, \frac{1}{2}) \rightarrow (2, -2.5)$

$(2, \frac{1}{4}) \rightarrow (4, -2.75)$

d) $y = -\frac{1}{10}(5)^{3x-9} + 10$ ← Factor this first to get the true horizontal translation.

$= -\frac{1}{10}(5)^{3(x-3)} + 10$

The base function is $y = 5^x$. It is being reflected over the x-axis, vertically compressed by a factor of $\frac{1}{10}$, horizontally compressed by a factor of $\frac{1}{3}$, translated 3 units to the right and 10 units up.



$(x, y) \longrightarrow \left(\frac{1}{3}x + 3, -\frac{1}{10}y + 10\right)$

$(-1, \frac{1}{5}) \longrightarrow (2\frac{2}{3}, 9\frac{49}{50})$

$(0, 1) \longrightarrow (3, 9\frac{9}{10})$

$(1, 5) \longrightarrow (3\frac{1}{3}, 9\frac{1}{2})$

12. y-int: $(0, 2)$

HA: $y = 1$

Possible eqn: $y = \left(\frac{1}{2}\right)^{-x} + 1$ which simplifies to $y = 2^x + 1$ since $\left(\frac{1}{2}\right)^{-x}$ is $(2^{-1})^{-x}$ or 2^x.

Handout #12

a) $2(3^{x-1}) = 54$

$3^{x-1} = 27$

$3^{x-1} = 3^3$

$\therefore x-1 = 3$

$\boxed{x = 4}$

b) $5^{x+1} + 5^{x+2} = 750$

$5^{x+1}(1+5^1) = 750$

$5^{x+1}(6) = 750$

$5^{x+1} = 125$

$5^{x+1} = 5^3$

$\therefore x+1 = 3$

$\boxed{x = 2}$

$\boxed{\text{OR}}$ $5^{x+1} + 5^{x+2} = 750$

$5^x(5+5^2) = 750$

$5^x(30) = 750$

$5^x = 25$

$5^x = 5^2$

$\therefore \boxed{x = 2}$

Handout #13 $\boxed{\text{Text}}$ p. 408 4.

4. In 2000, pop^n was 15,000, growing at a rate of 5%/yr.

$y = ab^x$

$y = 15\,000(1.05)^x$ where x is time in yrs since 2000

sub $x = 20$

$y = 15\,000(1.05)^{20}$

$\doteq 39{,}799$

$\therefore$ The best estimate for the pop^n in the yr 2020 is option (a), 40 000 people.

Handout #14

$P = \$4500$

$r = 0.035$

$i = \dfrac{0.035}{12}$

$n = 5 \times 12$

$n = 60$

$A = P(1+i)^n$

$= 4500\left(1 + \dfrac{0.035}{12}\right)^{60}$

$\doteq 5\,359.24$

$\therefore$ After 5yrs, the GIC would be worth \$5 359.24.

Handout #15

The unit circle should be in your notes for Unit 5.

Handout #16

a) A: 2.5 units

VT: 2.5 units up

P: 720°

$\therefore k = \frac{360°}{720°}$ or $\frac{1}{2}$

For a sine curve w P.S. 180° to the right:

$$y = 2.5\sin\left(\tfrac{1}{2}(\theta - 180°)\right) + 2.5$$

For a reflected cosine curve:

$$y = -2.5\cos\left(\tfrac{1}{2}\theta\right) + 2.5$$

or $y = -2.5\cos\left(\frac{\theta}{2}\right) + 2.5$

b) A: 2 cm

VT: 1 cm up

P: 10 seconds

$\therefore k = \frac{36\cancel{0}}{1\cancel{0}}$ or 36

For a sine curve with P.S. 7.5 units to the right:

$$h(t) = 2\sin(36(t - 7.5)) + 1$$

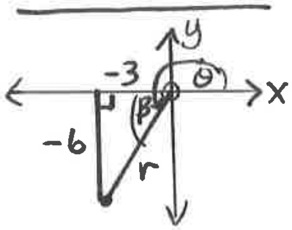
or For a reflected sine curve w P.S. 2.5 units right:

$$h(t) = -2\sin(36(t - 2.5)) + 1$$

For a cosine curve w no reflection or P.S.:

$$h(t) = 2\cos(36t) + 1$$

Handout #17



$r = \sqrt{x^2 + y^2}$

$= \sqrt{9 + 36}$

$= \sqrt{45}$

$= 3\sqrt{5}$

Exact Ratios

$\cos\theta = \frac{x}{r}$

$= \frac{-\cancel{3}^{-1}}{\cancel{3}\sqrt{5}}$

$= -\frac{\sqrt{5}}{5}$

$\sin\theta = \frac{y}{r}$

$= \frac{-\cancel{6}^{-2}}{\cancel{3}\sqrt{5}}$

$= \frac{-2\sqrt{5}}{5}$

$\tan\theta = \frac{y}{x}$

$= \frac{-6}{-3}$ or 2

$\theta = \frac{r}{x}$

$= \frac{\cancel{3}\sqrt{5}}{-\cancel{3}_{-1}}$

$= -\sqrt{5}$

$\csc\theta = \frac{r}{y}$

$= \frac{\cancel{3}\sqrt{5}}{-\cancel{6}_{-2}}$

$= -\frac{\sqrt{5}}{2}$

$\cot\theta = \frac{1}{2}$

Rotational Angle θ

$\beta = \tan^{-1}(2)$

$\doteq 63°$

$\therefore$ In Quad III

$\theta = 180° + 63°$

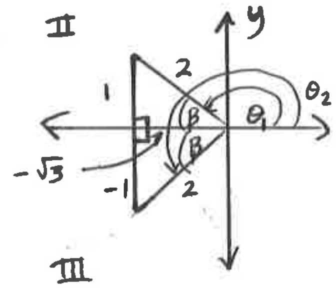
$\boxed{\theta \doteq 243°}$

Handout # 18

$0° \le A \le 360°$

a) $\cos A = -\frac{\sqrt{3}}{2}$

$\boxed{\beta = 30°}$



In Quad II

$\theta = 180° - 30°$
$= 150°$

In Quad III

$\theta = 180° + 30°$
$= 210°$

b) $\tan A = -1$ ← 45°, 45°, 90° △ where $x=1, y=-1$ or $x=-1, y=1$

In Quad II

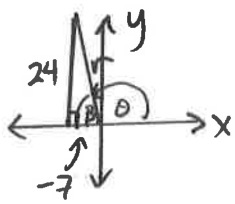
$\theta = 180° - 45°$
$= 135°$

In Quad IV

$\theta = 360° - 45°$
$= 315°$

Handout #19 $\boxed{\text{Text}}$ p.408 5–6, 10, 12, 13, 15–17

p.408 #5.



$x = -7, y = 24$

$\beta = \tan^{-1}\left(\frac{24}{7}\right)$

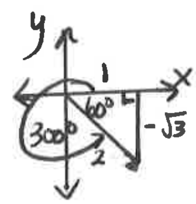
$\boxed{\beta \doteq 74°}$

$\theta = 180° - 74°$

$\boxed{\theta = 106°}$

∴ Answer is option (a).

#6. $\csc 300° = \frac{r}{y}$

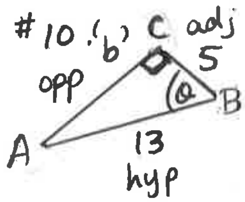


$= \frac{2}{-\sqrt{3}}$

$= \frac{-2\sqrt{3}}{3}$

∴ Option (c).

#10.



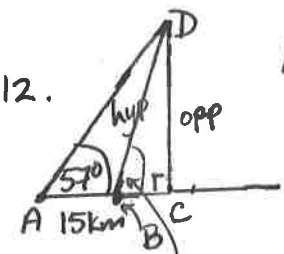
$\csc\theta = \frac{\text{hyp}}{\text{opp}}$

$= \frac{13}{12}$

∴ option (c)

$b^2 = c^2 - a^2$
$b^2 = 13^2 - 5^2$
$b^2 = 144$
$\boxed{b = 12 \text{ units}}$

#12.



$\angle DBC = 83°$

Let CD rep. the altitude of the weather balloon.

(i) $\angle ABD = 97°$ (SAT)
$\angle ADB = 26°$ (AST)

(ii) $\frac{BD}{\sin 57°} = \frac{15}{\sin 26}$

$\boxed{BD \doteq 28.7 \text{ km}}$

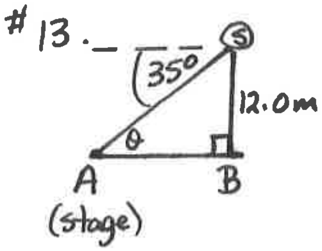
(iii) $\sin 83° = \frac{CD}{28.7}$

$CD \doteq 28.5$

∴ The altitude of the balloon is approx 28.5 km which is option (a).

Handout # 19 Cont'd $\boxed{\text{Text}}$ p. 408 13, 15–17

#13.



Let AS rep. the distance from the spotlight to the stage.

$\theta = 35°$ (AA)

$\sin\theta = \frac{opp}{hyp}$

$\sin 35° = \frac{12.0}{AS}$

$AS = \frac{12.0}{\sin 35°}$

$AS \doteq 20.9$

$\therefore$ The distance is approx 20.9 m which is option (a).

#15. $y = 2\cos 2(\theta + 45°) + 4$ is the graph of $y = \cos\theta$ vert. exp. by a factor of 2 (so the amplitude is 2), translated up 4 units (so the equ^n of the axis is $y=4$, min $y=2$ and max y is 6), horizontally compressed by a factor of $\frac{1}{2}$ (so the period is 180°) and with a P.S. to the left 45°.
Only the graph for (d) matches the above.

#16. $y = 2\cos 2\theta$ is the graph of $y = \cos\theta$ vert. exp. by a factor of 2 (max $y = 2$, min $y = -2$, $y_{avg} = 0$) and horiz. comp. by a factor of $\frac{1}{2}$ (period is 180°). This matches the graph for (a).

#17. sine function

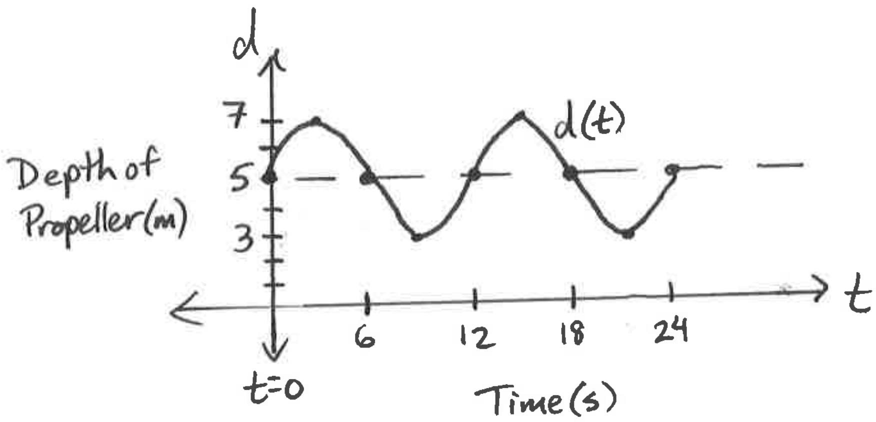
P: 720° $\therefore$ $k = \frac{360°}{720°}$ or $\frac{1}{2}$

If R: $\{y \in R \mid 2 \le y \le 12\}$ and A: 5, this means the equ^n of the axis is $y = 7$ (5 units away from both max and min y-values.

$\therefore$ The equ^n is $y = 5\sin\left(\frac{1}{2}\theta\right) + 7$ which matches (c).

Handout #20 $\boxed{\text{Text}}$ p. 404 # 6, 9, 10, 12, 13

#6. $d(t) = 2\sin(30t)^\circ + 5$



RW "roots"

$0^\circ \times \frac{1}{30} = 0$ sec

$\cancel{180}^{6}{}^\circ \times \frac{1}{\cancel{30}_1} = 6$ sec

$\cancel{360}^{12}{}^\circ \times \frac{1}{\cancel{30}_1} = 12$ sec

a) P: 12 sec. The period rep. the length of time between each wave.

b) If there were no waves the depth of the propeller would just be 5m.

c) $d(5.5) = 2\sin(30 \cdot 5.5)^\circ + 5$

$= 2\sin 165^\circ + 5$

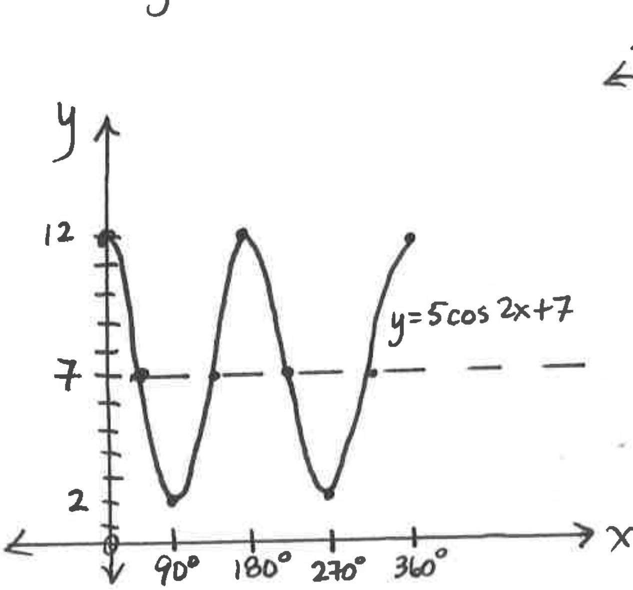
$\doteq 5.5$

At 5.5 s the depth of the propeller is approx 5.5 m.

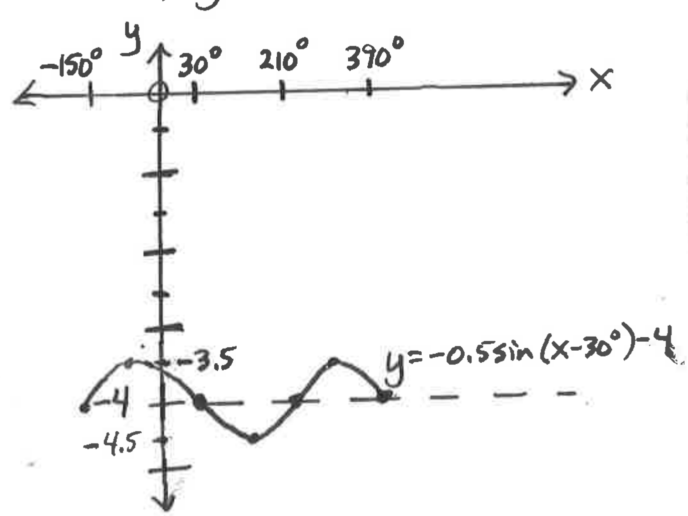
d) R: $\{d \in R \mid 3 \le d \le 7\}$

e) Within the first 10 seconds the only time the propeller is at a depth of 3m is when $t = 9$ seconds.

#9. a) $y = 5\cos 2x + 7$



9. b) $y = -0.5\sin(x - 30^\circ) - 4$



RW "roots"

$0^\circ + 30^\circ = 30^\circ$

$180^\circ + 30^\circ = 210^\circ$

$360^\circ + 30^\circ = 390^\circ$

- roots are still 180° apart
- reflected sine curve

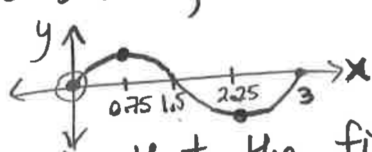
#10 a) VT : 2 units up and A: 3 units
$\therefore y_{max} = 5$ and $y_{min} = -1$ and $R: \{y \in \mathbb{R} \mid -1 \le y \le 5\}$

b) No V.T. and A: 0.5
$\therefore y_{max} = 0.5$ and $y_{min} = -0.5$ and $R: \{y \in \mathbb{R} \mid -0.5 \le y \le 0.5\}$

#12. a) $y_{max} = 3.5$, $y_{min} = 1.5$ $\therefore$ A: 1 and VT: 2.5 units up
Half of a cycle goes from $x = 1$ to $x = 7$ $\therefore P = 12$ and $K = \frac{\cancel{360}^{30}}{\cancel{12}_1}$
P.S. is 4 units to the right.

* $\boxed{\therefore \text{The equ}^{n} \text{ is } y = \sin(30(x-4))^\circ + 2.5}$

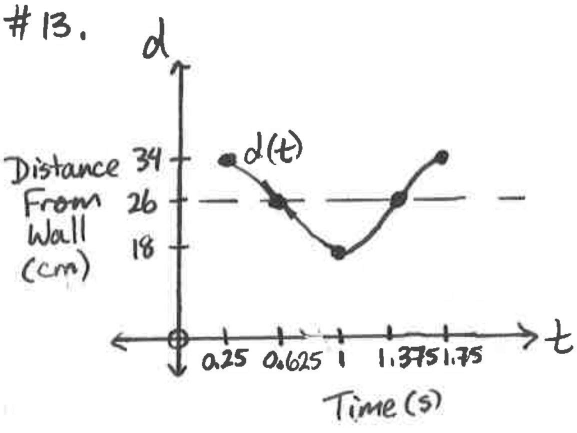
b) VT : 4 units up
A: 2 units
Half a cycle goes from $x = 3.5$ to $x = 5$ which is 1.5 units
$\therefore P = 3$ and $K = \frac{\cancel{360}^{120}}{\cancel{3}_1}$
The P.S. is tricky! If each cycle consists of 4 sections and the period is 3 units, then each section is 0.75 units apart.
i.e.



For the graph given in the text, note that the first crest or peak is at $x = 2$, so the usual starting point for a sine curve is 0.75 units to the left of this crest at $x = 1.25$.

* $\boxed{\therefore \text{The equ}^{n} \text{ is } y = 2\sin(120(x-1.25))^\circ + 4.}$

#13.



a) The period is 1.5 s and it rep. the amt. of time it takes to rock back and forth.

b) The chair is 26 cm from the wall when no one is rocking in it.

c) If Megan rocks back and forth 40 times then $D: \{t \in \mathbb{R} \mid 0 \le t \le 60\}$

d) $R: \{d \in \mathbb{R} \mid 18 \le d \le 34\}$

e) The amplitude is 8 cm and it rep. the max distance between the resting position and the back of the chair when it is forward or back from this position.

f) $d = 8\cos[240(t - 0.25)] + 26$
or $d = 8\cos[240(t - 1.75)] + 26$

RW
$K = \frac{360}{1.5}$
$= \cancel{360}^{120} \times \frac{2}{\cancel{3}}$
$= 240$

Handout #21 $\boxed{\text{Text}}$ p. 311 #14

a) (iii) is not an identity. It is not possible for L.S. = R.S.

b) (i) $L.S. = (1-\cos^2 x)(1-\tan^2 x)$

$= (1-\cos^2 x)\left(1-\dfrac{\sin^2 x}{\cos^2 x}\right)$

$= \sin^2 x\left(\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}\right)$

$= \dfrac{\sin^2 x\cos^2 x-\sin^4 x}{\cos^2 x}$

$= \dfrac{\sin^2 x(1-\sin^2 x)-\sin^4 x}{\cos^2 x}$

$= \dfrac{\sin^2 x-\sin^4 x-\sin^4 x}{\cos^2 x}$

$= \dfrac{\sin^2 x-2\sin^4 x}{1-\sin^2 x}$

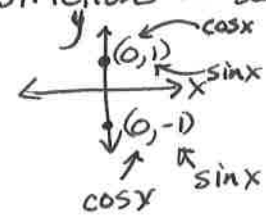
$R.S. = \dfrac{\sin^2 x-2\sin^4 x}{1-\sin^2 x}$

Restrictions:

$\because\ 1-\sin^2 x = (1+\sin x)(1-\sin x)$

$\therefore\ \sin x \neq \pm 1$

Note that $\cos x \neq 0$ provides the same restrictions as above



QED

(ii) $L.S. = 1-2\cos^2\phi$

$R.S. = \sin^4\phi-\cos^4\phi$

$= (\sin^2\phi+\cos^2\phi)(\sin^2\phi-\cos^2\phi)$

$= (1)(1-\cos^2\phi-\cos^2\phi)$

$= 1-2\cos^2\phi$

$\square$

No restrictions

(iv) $L.S. = \dfrac{1+2\sin\beta\cos\beta}{\sin\beta+\cos\beta}$

$R.S. = \sin\beta+\cos\beta$

$= \dfrac{\sin^2\beta+2\sin\beta\cos\beta+\cos^2\beta}{\sin\beta+\cos\beta}$

$= \dfrac{(\sin\beta+\cos\beta)^{\cancel{2}}}{\cancel{(\sin\beta+\cos\beta)}}$ cancel

$= \sin\beta+\cos\beta$

L.S. = R.S.

Restrictions

$\sin\beta \neq -\cos\beta$

14. b) (v) $L.S. = \dfrac{1-\cos\beta}{\sin\beta}$ $R.S. = \dfrac{\sin\beta}{1+\cos\beta}$

$$= \frac{(1-\cos\beta)}{\sin\beta}\cdot\frac{(1+\cos\beta)}{(1+\cos\beta)}$$

$$= \frac{1-\cos^2\beta}{\sin\beta(1+\cos\beta)}$$

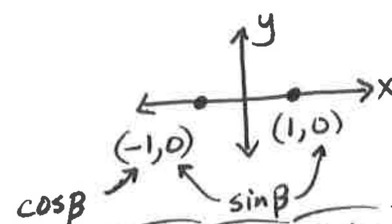
$$= \frac{\sin^{\cancel{2}}\beta}{\cancel{\sin\beta}(1+\cos\beta)} \quad \text{cancel!}$$

$$= \frac{\sin\beta}{1+\cos\beta}$$

$\square$

Restrictions:

$\sin\beta \neq 0$



Note that $\sin\beta \neq 0$ includes $\cos\beta \neq -1$

(vi) $L.S. = \dfrac{\sin x}{1+\cos x}$

$R.S. = \csc x - \cot x$

$$= \frac{1}{\sin x} - \frac{\cos x}{\sin x}$$

$$= \frac{1-\cos x}{\sin x}$$

Note that at this point it becomes the same Q as (v) above on the L.S.

$$= \frac{(1-\cos x)}{\sin x}\cdot\frac{(1+\cos x)}{(1+\cos x)}$$

$$= \frac{1-\cos^2 x}{\sin x\,(1+\cos x)}$$

$$= \frac{\sin^{\cancel{2}} x}{\cancel{\sin x}(1+\cos x)}$$

$$= \frac{\sin x}{1+\cos x}$$

QED

Restrictions: $\sin x \neq 0$