MCR3U

Lessons 7 -8: Sketching Functions Using Transformations

Problem Set

1. Provide a labeled sketch of the graph of $f(x) = -3x^2 - 2x + 3$. Include the exact location of the vertex and approximate locations of all intercepts.

2. Suppose that (-2, 9) is a point on the graph of $y = f(x)$. For each of the following describe the transformations involved, provide the mapping notation and give the coordinates of a point on the graph.

   a) $y = 2f(x)$ b) $g(x) = -\frac{1}{2}f(x)$ c) $y = f(2x)$ d) $y = f\left(\frac{x}{4}\right)$ e) $g(x) = -f(4x)$

3. Suppose that (1, 5) is a point on the graph of $y = f(x)$. Give the coordinates of a point on the graph of:

   a) $y = f(x - 5)$ b) $g(x) = f(x) + 4$ c) $y = f(x + 1) - 9$

4. Text page 70 #3, 6bc

5. Let $f(x) = \sqrt{x}$ and $g(x) = -2\sqrt{-x}$.

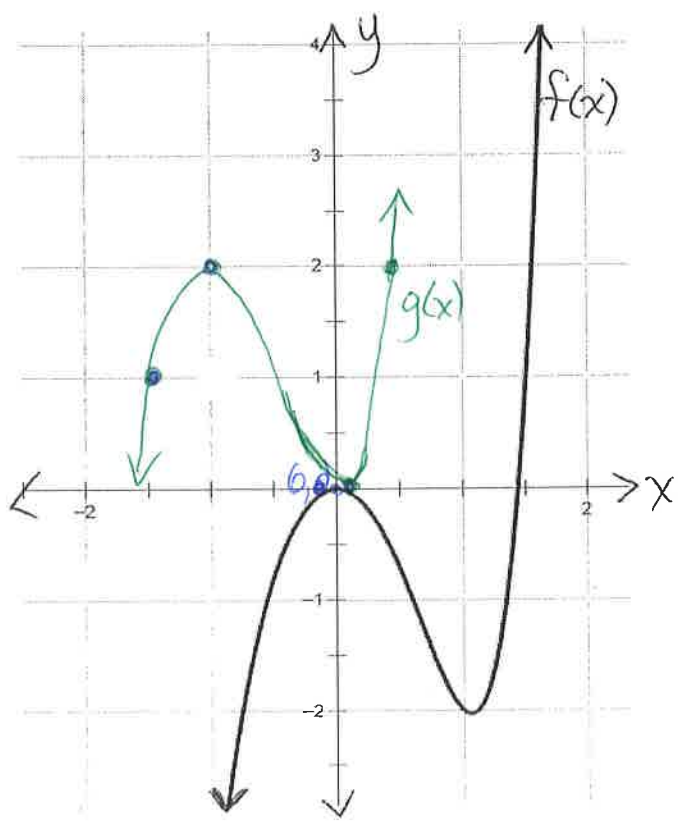
   a) Sketch f(x) and g(x). (Label key points).
   b) Find the domain and range of g(x).
   c) Write g(x) in terms of f(x).

6. Let $f(x) = |x|$ and $g(x) = |x - 3| + 5$.

   a) Sketch f(x) and g(x) (Label key points).
   b) Find the domain and range of g(x).

7. Let *f(x)* be the graph shown below to the right. Let $g(x) = f(x + 1) + 2$.

   a) Sketch the graph of g(x). (You may estimate some of the points on f(x)).
   b) The equation for this graph is in fact $f(x) = x^5 - 3x^2$. Given this fact, what is the equation for g(x)? (You do not need to simplify this equation).



8. Let $f(x) = |x|$ and $g(x) = -2|-(x+2)| + 5$. Sketch the graph of g(x).

9. Let $f(x) = \sqrt{x}$ and $g(x) = -2f[0.5(x+3)] + 3$.

   a) Sketch the graph of *g(x)*.
   b) Find the domain and range of *g(x)*.
   c) Write the " specific equation" for *g(x)*. Then, use your equation to verify that one the points on your graph of *g(x)* is correct.

10. Let $g(x) = \frac{-2}{x-4} + 3$. Sketch the graph of *g(x)*. Include all key points and asymptotes.

11. Let $f(x) = \frac{5}{x+2} - 8$.
   a) Find the domain and range of *f(x)*. (You do not need to sketch the function to do this).
   b) Find the equations of any asymptotes of *f(x)*.

12. Text page 71 #12, 20

13. Let $f(x) = \frac{1}{x^2+1}$ and $g(x) = -2f[(x-3)] + 2$. Sketch *g(x)*.

Answers

For questions requiring a sketch, see the answers posted in the classroom i.e. for #1, 5a, 6a, 8, 9a, 10, 13

#2. a) vert. exp. factor of 2; $(x,y) \to (x,2y)$; $(-2,18)$
b) vert. comp. factor of $\frac{1}{2}$ and reflected over x-axis; $(x,y) \to (x,-\frac{1}{2}y)$, $(-2,-\frac{9}{2})$
c) horiz. comp. factor of $\frac{1}{2}$; $(x,y) \to (\frac{1}{2}x,y)$; $(-1,9)$
d) horiz. exp. factor of 4; $(x,y) \to (4x,y)$; $(-8,9)$
e) refl. over x-axis and horiz. comp. factor of $\frac{1}{4}$; $(x,y) \to (\frac{1}{4}x,-y)$; $(-\frac{1}{2},-9)$

3. a) $(6,5)$ b) $(1,9)$ c) $(0,-4)$

5. b) D: $\{x \in R \mid x \le 0\}$ R: $\{y \in R \mid y \le 0\}$ c) $g(x) = -2f(-x)$

6. b) D: $\{x \in R\}$ R: $\{y \in R \mid y \ge 5\}$

7. b) $g(x) = (x+1)^5 - 3(x+1)^3 + 2$

9. b) D: $\{x \in R \mid x \ge -3\}$ R: $\{y \in R \mid y \le 3\}$ c) $g(x) = -2\sqrt{0.5(x+3)} + 3$

11. a) D: $\{x \in R \mid x \ne -2\}$ R: $\{y \in R \mid y \ne -8\}$ b) VA: $x = -2$, HA: $y = 3$

L7-8 Sketching Functions Using Transformations – Problem Set

Day 1 #1–4

#1. $f(x) = -3x^2 - 2x + 3$

$= -3\left[\left(x^2 + \frac{2}{3}x + \frac{1}{9}\right) - \frac{1}{9}\right] + 3$

$= -3\left(x + \frac{1}{3}\right)^2 + \frac{1}{3} + \frac{9}{3}$

$= -3\left(x + \frac{1}{3}\right)^2 + \frac{10}{3}$

$\therefore V\left(-\frac{1}{3}, \frac{10}{3}\right)$

y-int: $(0,3)$

x-int, $f(x) = 0$

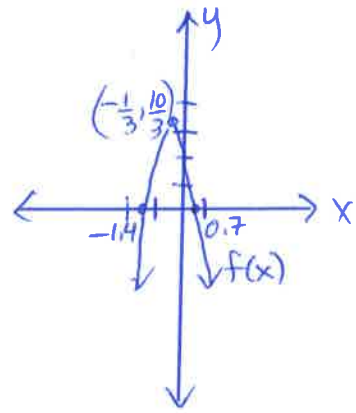
$0 = -3x^2 - 2x + 3$

$0 = -(3x^2 + 2x - 3)$

$x = \frac{-2 \pm \sqrt{4+36}}{6}$

$= \frac{-2 \pm \sqrt{40}}{6}$

$\doteq 0.72$ and -1.39



2. $(-2, 9)$ is a point on $y = f(x)$.

a) If $y = 2f(x)$ the graph is vertically expanded by a factor of 2 so the point is $(-2, 18)$ i.e. $(x,y) \to (x, 2y)$

b) If $g(x) = -\frac{1}{2}f(x)$, the graph is vert. compressed by a factor of $\frac{1}{2}$ and reflected over the x-axis, i.e. $(x,y) \to \left(x, -\frac{1}{2}y\right)$ so the pt. is $\left(-2, -\frac{9}{2}\right)$

c) If $y = f(2x)$, the graph is horiz. comp. by a factor of $\frac{1}{2}$ so $(x,y) \to \left(\frac{1}{2}x, y\right)$ and the pt. is $(-1, 9)$.

d) If $y = f\left(\frac{x}{4}\right)$, the graph is horiz. expanded by a factor of 4. $(x,y) \to (4x, y)$ and the pt. is $(-8, 9)$.

e) If $g(x) = -f(4x)$ the graph is refl. over the x-axis and horiz. comp. by a factor of $\frac{1}{4}$. $(x,y) \to \left(\frac{1}{4}x, -y\right)$ and the pt. is $\left(-\frac{1}{2}, -9\right)$.

3. $(1, 5)$ is a pt. on $y = f(x)$.

a) If $y = f(x-5)$ then the pt. is $(6, 5)$ ← translation 5 units to the right

b) If $g(x) = f(x) + 4$ then the pt. is $(1, 9)$

c) If $y = f(x+1) - 9$ then the pt. is $(0, -4)$ ← transl. left 1 unit, down 9 units

→

4. Text. p 70

#3. Given $(1,1)$

$f(x)$	$f(3x)$	$f(-3x)$	$5f(-3x)$	$5f(-3(x-2))+4$
$(1,1)$	$\left(\frac{1}{3},1\right)$	$\left(-\frac{1}{3},1\right)$	$\left(-\frac{1}{3},5\right)$	$(x,y)\rightarrow\left(-\frac{1}{3}x+2,5y+4\right)$ $(1,1)\rightarrow\left(\frac{5}{3},9\right)$

6. b) $y=2f(-(x-3))+1$

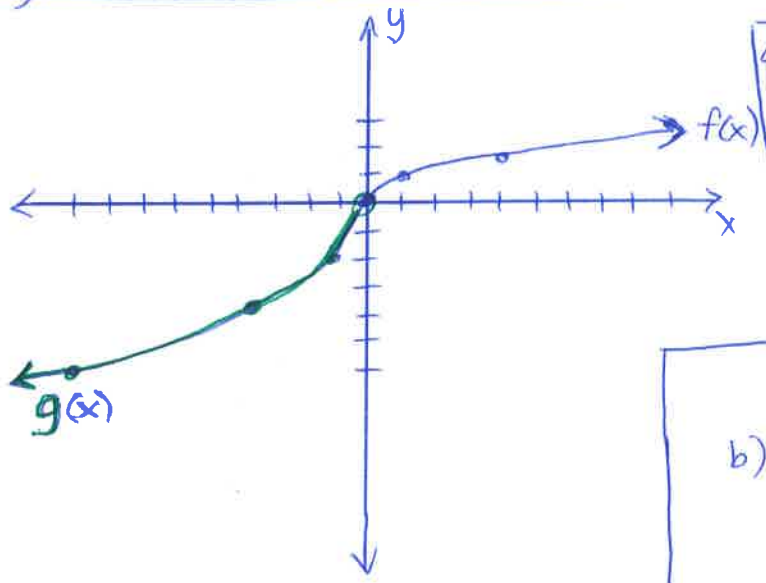
- horiz. transl. 3 units right
- refl. in y-axis
- vert. exp. factor of 2
- transl. up 1 unit

c) $y=-3f(2(x-1))-3$

- horiz. comp. factor of $\frac{1}{2}$
- transl. 1 unit right
- refl. in x-axis
- vert exp. factor of 3
- transl. down 3 units

L7-8 Sketching Functions Using Transformations Day 2 #5-13

5.



a) $f(x)=\sqrt{x}$ $g(x)=-2\sqrt{-x}$

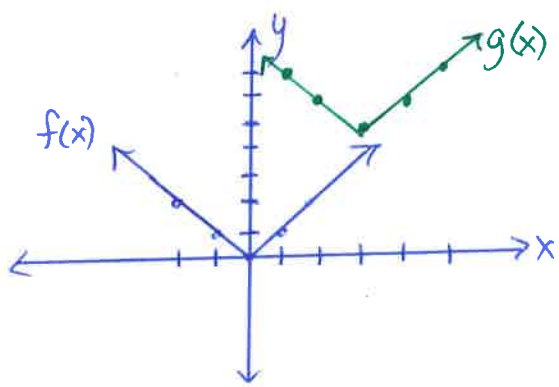
$(x,y) \rightarrow (-x,-2y)$
$(0,0) \rightarrow (0,0)$
$(1,1) \rightarrow (-1,-2)$
$(4,2) \rightarrow (-4,-4)$
$(9,3) \rightarrow (-9,-6)$

b) for g(x) D: $\{x \in R \mid x \le 0\}$
R: $\{y \in R \mid y \le 0\}$

c) $g(x) = -2f(-x)$

6. $f(x)=|x|$ $g(x)=|x-3|+5$

a) $(x,y) \rightarrow (x+3, y+5)$
$(-2,2) \rightarrow (1,7)$
$(-1,1) \rightarrow (2,6)$
$(0,0) \rightarrow (3,5)$
$(1,1) \rightarrow (4,6)$
$(2,2) \rightarrow (5,7)$



b) for g(x) D: $\{x \in R\}$ R: $\{y \in R \mid y \ge 5\}$

7. a) $g(x) = f(x+1)+2$
$(x,y) \rightarrow (x-1, y+2)$
$(-0.5,-1) \rightarrow (-1.5,1)$
$(0,0) \rightarrow (-1,2)$
$(1.1,-2) \rightarrow (0.1,0)$
$(1.4,0) \rightarrow (0.4,2)$

See Question Sheet for Graph

b) If $f(x) = x^5-3x^2$
then $g(x) = (x+1)^5-3(x+1)^3+2$

8. $f(x) = |x| \quad g(x) = -2|-(x+2)| + 5$

$(x,y) \rightarrow (-x-2, -2y+5)$

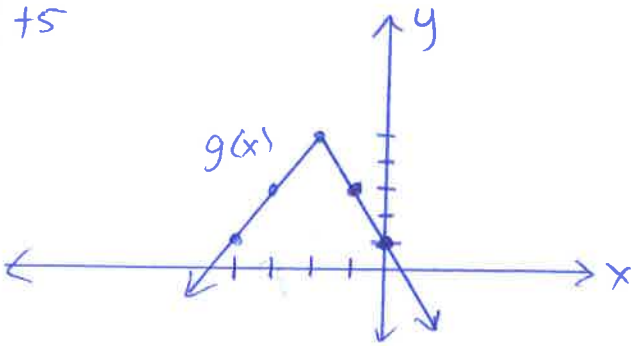
$(-2,2) \rightarrow (0,1)$

$(-1,1) \rightarrow (-1,3)$

$(0,0) \rightarrow (-2,5)$

$(1,1) \rightarrow (-3,3)$

$(2,2) \rightarrow (-4,1)$



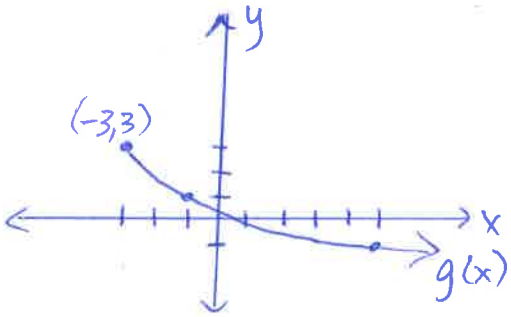
9. a) $f(x) = \sqrt{x} \quad g(x) = -2f[0.5(x+3)] + 3$

$(x,y) \rightarrow (2x-3, -2y+3)$

$(0,0) \rightarrow (-3,3)$

$(1,1) \rightarrow (-1,1)$

$(4,2) \rightarrow (5,-1)$



b) D: $\{x \in R \mid x \geq -3\}$ R: $\{y \in R \mid y \leq 3\}$

c) Specific equ^{n}: $g(x) = -2\sqrt{0.5(x+3)} + 3$

test $(5,-1)$

$g(5) = -2\sqrt{0.5(5+3)} + 3$

$= -2\sqrt{0.5(8)} + 3$

$= -2\sqrt{4} + 3$

$= -2(2) + 3$

$= -4 + 3$

$= -1$ ✓

10. $g(x) = \dfrac{-2}{x-4} + 3$

HA: $y = 3$

VA: $x = 4$

$(x,y) \rightarrow (x+4, -2y+3)$

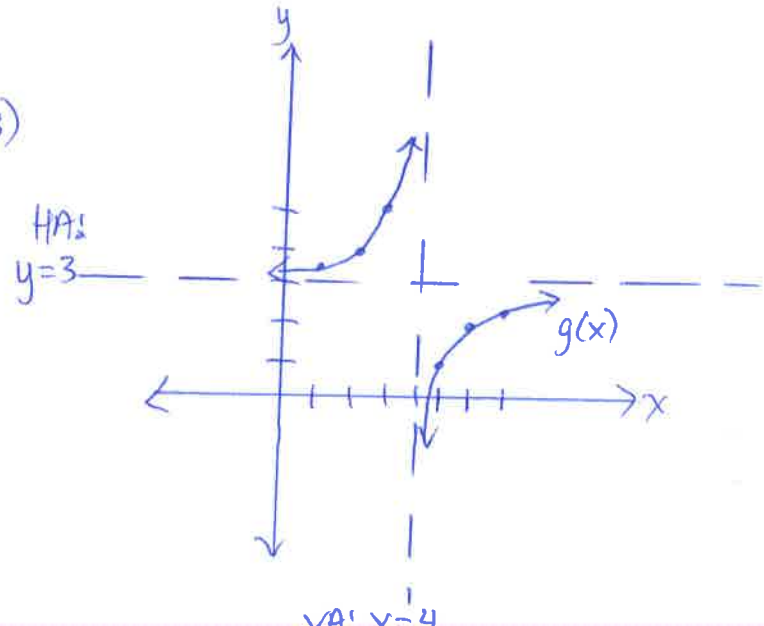
$(-2, -\frac{1}{2}) \rightarrow (2, 4)$

$(-1,-1) \rightarrow (3, 5)$

$(1,1) \rightarrow (5,1)$

$(2, \frac{1}{2}) \rightarrow (6,2)$

$(3, \frac{1}{3}) \rightarrow (7, \frac{7}{3})$



11. $f(x) = \frac{5}{x+2} - 8$

a) D: $\{x \in R \mid x \neq -2\}$ R: $\{y \in R \mid y \neq -8\}$ b) VA: $x = -2$, HA: $y = -8$

12. $f(x) = \sqrt{x}$, $h(x) = f(-3x-12)$

Text p71 #12

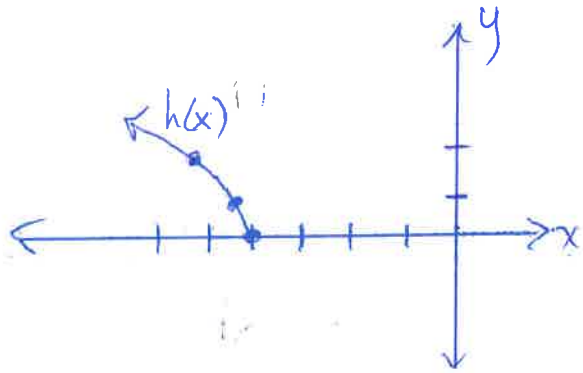
$= f(-3(x+4))$

$(x,y) \rightarrow (-\frac{1}{3}x - 4, y)$

$(0,0) \rightarrow (-4,0)$

$(1,1) \rightarrow (-\frac{13}{3}, 1)$

$(4,2) \rightarrow (-\frac{16}{3}, 2)$



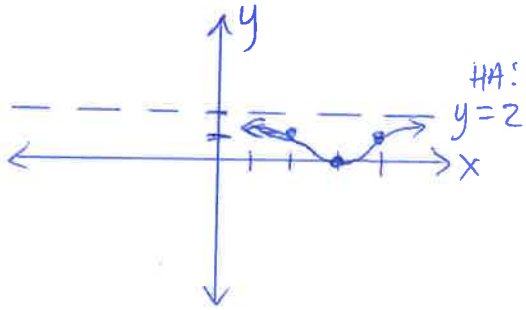
p71 20. If $f(x) = (x-2)(x+5)$ find the x-intercepts for:

a) $y = f(x)$	b) $y = -4f(x)$	c) $y = f(-\frac{1}{3}x)$	d) $y = f(-(x+2))$
$(2,0)$ $(-5,0)$	still $(2,0)$ $(-5,0)$	$(-6,0)$ $(15,0)$	$(-4,0)$ $(3,0)$

13. $f(x) = \frac{1}{x^2+1}$ $g(x) = -2f[(x-3)] + 2$ ← HA: $y = 2$

$(x,y) \rightarrow (x+3, -2y+2)$

$(-1, \frac{1}{2}) \rightarrow (2, 1)$

$(0,1) \rightarrow (3,0)$

$(1, \frac{1}{2}) \rightarrow (4, 1)$