Rational Expressions and Applications

Show that the difference between reciprocals of consecutive positive integers is the reciprocal of their product.

consecutive pos. integers : 3, 4 so the recip. are $\frac{1}{3}$, $\frac{1}{4}$ and recip. of their product is $\frac{1}{12}$

$$\frac{1}{3} - \frac{1}{4} = \frac{1}{12} \text{ is true } \therefore \frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$$

For the general case, let n and $n+1$ rep. the consecutive integers.

$$\frac{1}{n} - \frac{1}{n+1}$$

$$= \frac{\cancel{n}+1 - \cancel{n}}{n(n+1)}$$

$$= \frac{1}{n(n+1)} \leftarrow \text{reciprocal of their product}$$

Text page 129 #9 page 114 #17 and page 133 #15, and page 134 #9

↑all parts 15 a–f

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9. a) $\frac{\cancel{2x}^{\cancel{3}} x^2}{\cancel{3y^2}} \times \frac{\cancel{9y}^3}{\cancel{10x}_5} - \frac{2y}{3x}$

$$= \frac{3x^2}{5y} - \frac{2y}{3x}$$

$$= \frac{9x^3 - 10y^2}{15xy}$$

$x, y \neq 0$

b) $\frac{x+1}{2x-6} \div \frac{2(x+1)^2}{2-x} + \frac{11}{x-2}$

$$= \frac{\cancel{x+1}}{2(x-3)} \times \frac{-(x-2)}{2(x+1)^{\cancel{2}}} + \frac{11}{x-2}$$

$$= \frac{-(x-2)}{4(x-3)(x+1)} + \frac{11}{x-2}$$

$$= \frac{-(x-2)^2 + 11(4)(x-3)(x+1)}{4(x-3)(x+1)(x-2)}$$

$$= \frac{-(x^2-4x+4) + 44(x^2-2x-3)}{4(x-3)(x+1)(x-2)}$$

$$= \frac{-x^2+4x-4+44x^2-88x-132}{4(x-3)(x+1)(x-2)}$$

$$= \frac{43x^2-84x-136}{4(x-3)(x+1)(x-2)}$$

$x \neq -1, 2, 3$

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9. c) $\frac{p+1}{p^2+2p-35} + \frac{p^2+p-12}{p^2-2p-24} \times \frac{p^2-4p-12}{p^2+2p-15}$

$= \frac{p+1}{(p+7)(p-5)} + \frac{\cancel{(p+4)}\cancel{(p-3)}}{\cancel{(p-6)}\cancel{(p+4)}} \times \frac{\cancel{(p-6)}(p+2)}{(p+5)\cancel{(p-3)}}$

$= \frac{p+1}{(p+7)(p-5)} + \frac{p+2}{p+5}$

$= \frac{(p+1)(p+5) + (p+2)(p+7)(p-5)}{(p+7)(p-5)(p+5)}$

$= \frac{p^2+6p+5 + (p+2)(p^2+2p-35)}{(p+7)(p-5)(p+5)}$

$= \frac{p^2+6p+5+p^3+2p^2-35p+2p^2+4p-70}{(p+7)(p-5)(p+5)}$

$= \frac{p^3+5p^2-25p-65}{(p+7)(p-5)(p+5)}$ $\quad p \neq -7, -5, -4, 3, 5, 6$

d) $\frac{5m-n}{2m+n} - \frac{4m^2-4mn+n^2}{4m^2-n^2} \div \frac{6m^2-mn-n^2}{3m+15n}$

$= \frac{5m-n}{2m+n} - \frac{(2m-n)^2}{(2m-n)(2m+n)} \div \frac{(3m+n)(2m-n)}{3(m+5n)}$

$= \frac{5m-n}{2m+n} - \frac{\cancel{(2m-n)}}{(2m+n)} \times \frac{3(m+5n)}{(3m+n)\cancel{(2m-n)}}$

$= \frac{5m-n}{2m+n} - \frac{3(m+5n)}{(2m+n)(3m+n)}$

$= \frac{(5m-n)(3m+n) - 3(m+5n)}{(2m+n)(3m+n)}$

Text error they have $-2mn$

$= \frac{15m^2+2mn-n^2-3m-15n}{(2m+n)(3m+n)}$ $\quad m \neq -\frac{1}{2}n, \frac{1}{2}n, -5n, -\frac{1}{3}n$

p.114 #17. a) $a(t) = \dfrac{-2(1+t^2)^2 + 2t(2)(1+t^2)(2t)}{(1+t^2)^4}$

$$= \frac{-2(1+t^2)^2 + 8t^2(1+t^2)}{(1+t^2)^4}$$

$$= \frac{-2(1+\cancel{t^2})[(1+t^2) - 4t^2]}{(1+t^2)^{\cancel{4}3}}$$

$$= \frac{-2(1-3t^2)}{(1+t^2)^3}$$

no restrictions

$$= \frac{2(3t^2-1)}{(1+t^2)^3}$$

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b) $f(x) = \dfrac{2(2x+1)(2)(3x-2)^3 - (2x+1)^2(3)(3x-2)^2}{(3x-2)^6}$

$$= \frac{4(2x+1)(3x-2)^3 - 3(2x+1)^2(3x-2)^2}{(3x-2)^6}$$

$$= \frac{(2x+1)\cancel{(3x-2)^2}[4(3x-2) - 3(2x+1)]}{(3x-2)^{\cancel{6}4}}$$

$$= \frac{(2x+1)(6x-11)}{(3x-2)^4}$$

$x \neq \frac{2}{3}$

p.133 #15.

a) $\dfrac{1}{2x} - \dfrac{7}{3x^2} + \dfrac{4}{x^3}$

$= \dfrac{3x^2 - 14x + 24}{6x^3}$

$x \neq 0$

b) $\dfrac{3x}{x+2} + \dfrac{4x}{x-6}$

$= \dfrac{3x^2 - 18x + 4x^2 + 8x}{(x+2)(x-6)}$

$= \dfrac{7x^2 - 10x}{(x+2)(x-6)}$ $\quad x \neq -2, 6$

c) $\dfrac{6x}{x^2 - 5x + 6} - \dfrac{3x}{x^2 + x - 12}$

$= \dfrac{6x}{(x-3)(x-2)} - \dfrac{3x}{(x+4)(x-3)}$

$= \dfrac{6x(x+4) - 3x(x-2)}{(x-3)(x-2)(x+4)}$

$= \dfrac{6x^2 + 24x - 3x^2 + 6x}{(x-3)(x-2)(x+4)}$

$= \dfrac{3x^2 + 30x}{(x-3)(x-2)(x+4)}$ $\quad x \neq -4, 2, 3$

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15. d) $\dfrac{2(x-2)^2}{x^2+6x+5} \times \dfrac{3x+15}{(2-x)^2}$

$= \dfrac{2(x-2)^2}{\cancel{(x+5)}(x+1)} \times \dfrac{3\cancel{(x+5)}}{(2-x)(2-x)}$

$= \dfrac{6\cancel{(x-2)^2}}{(x+1)} \times \dfrac{1}{(-1)\cancel{(x-2)}(-1)\cancel{(x-2)}}$

$= \dfrac{6}{x+1} \qquad x \neq -5, -1, 2$

e) $\dfrac{(x-2y)^2}{x^2-y^2} \div \dfrac{(x-2y)(x+3y)}{(x+y)^2}$

$= \dfrac{(x-2y)^{\cancel{2}}}{(x-y)\cancel{(x+y)}} \times \dfrac{(x+y)^{\cancel{2}}}{\cancel{(x-2y)}(x+3y)}$

$= \dfrac{(x-2y)(x+y)}{(x-y)(x+3y)} \qquad x \neq \pm y, -3y, 2y$

f) $\dfrac{2b-5}{b^2-2b-15} + \dfrac{3b}{b^2+b-30} \times \dfrac{b^2+8b+12}{b+3}$

$= \dfrac{2b-5}{(b-5)(b+3)} + \dfrac{3b}{\cancel{(b+6)}(b-5)} \times \dfrac{\cancel{(b+6)}(b+2)}{(b+3)}$

$= \dfrac{2b-5}{(b-5)(b+3)} + \dfrac{3b^2+6b}{(b-5)(b+3)}$ same denom!

$= \dfrac{3b^2+8b-5}{(b-5)(b+3)} \qquad b \neq -3, -6, 5$

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9. Roman's conjecture is that for the sum of the reciprocals of 3 consec. natural numbers, the numerator is 3x product of 1st and third denom. plus 2, and the denom. of the sum is the product of the three denom.

Given

$\frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

Applying the rule

$\text{sum} = \dfrac{3(3)(5)+2}{3(4)(5)}$

$= \dfrac{45+2}{60}$

$= \dfrac{47}{60}$

$\frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

Apply Rule

$\text{sum} = \dfrac{3(4)(6)+2}{4(5)(6)}$

$= \dfrac{74}{120}$

$= \dfrac{37}{60}$

General Rule: Let $n, n+1, n+2$ rep. the three consec. natural numbers.

$\dfrac{1}{n} + \dfrac{1}{n+1} + \dfrac{1}{n+2}$

$\text{sum} = \dfrac{1(n+1)(n+2) + n(n+2) + n(n+1)}{n(n+1)(n+2)}$

$= \dfrac{n^2+3n+2+n^2+2n+n^2+n}{n(n+1)(n+2)}$

$= \dfrac{3n^2+6n+2}{n(n+1)(n+2)}$ Factor to leave +2

$= \dfrac{3n(n+2)+2}{n(n+1)(n+2)}$

∴ Roman is correct.