

U2/ Review

$$1. a) 5x^2 + 3xy - 2y^2 \\ = (5x - 2y)(x + y)$$

$$b) y^2 - 4x^2 - 4x - 1 \\ = y^2 - (4x^2 + 4x + 1) \\ = y^2 - (2x + 1)^2 \\ = (y + 2x + 1)(y - 2x - 1) \\ = (2x + y + 1)(-2x + y - 1) \\ = -(2x + y + 1)(2x - y + 1)$$

$$c) n^3 + 3n^2 + 2n + 6 \\ = n^2(n + 3) + 2(n + 3) \\ = (n^2 + 2)(n + 3)$$

2. x-int, f(x) = 0

$$0 = 4x^2(x + 3) - (x + 3)$$

$$0 = (4x^2 - 1)(x + 3)$$

$$0 = (2x + 1)(2x - 1)(x + 3)$$

$$\therefore x = \pm \frac{1}{2}, -3$$

3. Text p.132 4, 6g, 7-10, 12-15 p.134 5-7

$$\#4. a) -3(7x - 5)(4x - 7) \\ = -3(28x^2 - 69x + 35) \\ = -84x^2 + 207x - 105$$

$$b) -(y^2 - 4y + 7)(3y^2 - 5y - 3) \\ = -(3y^4 - 5y^3 - 3y^2 - 17y^3 + 20y^2 + 12y + 21y^2 - 35y - 21) \\ = -3y^4 + 17y^3 - 38y^2 + 23y + 21$$

$$c) 2(a + b)^3 \\ = 2(a^3 + 3a^2b + 3ab^2 + b^3) \\ = 2a^3 + 6a^2b + 6ab^2 + 2b^3$$

$$d) 3(x^2 - 2)^2(2x - 3)^2 \\ = 3(x^4 - 4x^2 + 4)(4x^2 - 12x + 9) \\ = 3(4x^6 - 12x^5 + 9x^4 - 16x^4 + 48x^3 - 36x^2 + 16x^2 - 48x + 36) \\ = 3(4x^6 - 12x^5 - 7x^4 + 48x^3 - 20x^2 - 48x + 36) \\ = 12x^6 - 36x^5 - 21x^4 + 144x^3 - 60x^2 - 144x + 108$$

$$6.g) (x^2 + 5x - 3)^2 \\ = (x^2 + 5x - 3)(x^2 + 5x - 3) \\ = x^4 + 5x^3 - 3x^2 + 5x^3 + 25x^2 - 15x - 3x^2 - 15x + 9 \\ = x^4 + 10x^3 + 19x^2 - 30x + 9$$

$$7. a) 12m^2n^3 + 18m^3n^2 \\ = 6m^2n^2(2n + 3m)$$

$$b) x^2 - 9x + 20 \\ = (x - 5)(x - 4)$$

$$c) 3x^2 + 24x + 45 \\ = 3(x^2 + 8x + 15) \\ = 3(x + 5)(x + 3)$$

$$d) 50x^2 - 72 \\ = 2(25x^2 - 36) \\ = 2(5x + 6)(5x - 6)$$

$$e) 9x^2 - 6x + 1 \\ = (3x - 1)^2$$

$$f) 10a^2 + a - 3 \\ = 10a^2 - 5a + 6a - 3 \\ = 5a(2a - 1) + 3(2a - 1) \\ = (5a + 3)(2a - 1)$$

$$8. a) 2x^2y^4 - 6x^5y^3 + 8x^3y$$

$$= 2x^2y(y^3 - 3x^3y^2 + 4x)$$

$$b) 2x(x+4) + 3(x+4)$$

$$= (2x+3)(x+4)$$

$$c) x^2 - 3x - 10$$

$$= (x-5)(x+2)$$

$$d) 15x^2 - 53x + 42$$

$$= (5x-6)(3x-7)$$

$$e) a^4 - 16$$

$$= (a^2+4)(a^2-4)$$

$$= (a^2+4)(a+2)(a-2)$$

$$f) (m-n)^2 - (2m+3n)^2$$

$$= [m-n - (2m+3n)][m-n + 2m+3n]$$

$$= (-m-4n)(3m+2n)$$

$$= -(m+4n)(3m+2n)$$

1	3	5	15	1	2	3	6
15	5	3	1	42	21	14	7

sum of 53

$$9. a) \frac{10a^2b + 15bc^2}{-5b}$$

$$= -2a^2 - 3c^2$$

$b \neq 0$

$$b) \frac{30x^2y^3 - 20x^2z^2 + 50x^2}{10x^2}$$

$$= 3y^3 - 2z^2 + 5$$

$x \neq 0$

$$c) \frac{xy - xyz}{xy}$$

$$= 1 - z$$

$x, y \neq 0$

$$d) \frac{16mnr - 24map + 40kmn}{8mn}$$

$$= 2r - 3p + 5k \quad m, n \neq 0$$

* 10. a) *Simplify first.*

$$8xy^2 + 12x^2y - \frac{6x^3z^2}{2xy}$$

$$= 8xy^2 + 12x^2y - \frac{3x^2z^2}{y}$$

$$= \frac{8xy^3 + 12x^2y^2 - 3x^2z^2}{y}$$

$y \neq 0$

$$b) \frac{7a-14b}{2(a-2b)}$$

$$= \frac{7(a-2b)}{2(a-2b)}$$

$$= \frac{7}{2}$$

$a \neq 2b$

$$c) \frac{m+3}{m^2+10m+21}$$

$$= \frac{m+3}{(m+3)(m+7)}$$

$$= \frac{1}{m+7} \quad m \neq -3, -7$$

$$d) \frac{4x^2 - 4x - 3}{4x^2 - 9}$$

$$= \frac{(2x+1)(2x-3)}{(2x+3)(2x-3)}$$

$$= \frac{2x+1}{2x+3} \quad x \neq \pm \frac{3}{2}$$

$$e) \frac{3x^2 - 21x}{7x^2 - 28x + 21}$$

$$= \frac{3x(x-7)}{7(x-1)(x-3)}$$

$x \neq 1, 3$

$$f) \frac{3x^2 - 2xy - y^2}{3x^2 + 4xy + y^2}$$

$$= \frac{(3x+y)(x-y)}{(3x+y)(x+y)}$$

$$= \frac{x-y}{x+y} \quad x = -\frac{1}{3}y, -y$$

P. 133

$$12. a) \frac{6x}{48y} \times \frac{1}{3x} \times \frac{1}{2y^2} = \frac{1}{2} \quad x, y \neq 0$$

$$b) \frac{10m^2}{18n} \times \frac{1}{20m^2} = m \quad m, n \neq 0$$

$$c) \frac{2ab}{5bc} \div \frac{3}{10b} = \frac{2a}{5c} \times \frac{5b}{3ac} = \frac{10b}{15c^2} \quad a, b, c \neq 0$$

$$d) \frac{5p}{8q} \div \frac{1}{3p} = \frac{5}{8q} \times \frac{3p}{1} = \frac{5}{2p} \quad p, q \neq 0$$

$$13. a) \frac{x^2}{2xy} \times \frac{1}{2y^2} \div \frac{(3x)^2}{xy^2} = \frac{x^2}{4y^3} \times \frac{1}{2y^2} \times \frac{xy^2}{9x^2} = \frac{x}{36y} \quad x, y \neq 0$$

$$b) \frac{x^2-5x+6}{x^2-1} \times \frac{x^2-4x-5}{x^2-4} \div \frac{x-5}{x^2+3x+2} = \frac{(x-3)(x-2)}{(x+1)(x-1)} \times \frac{(x-5)(x+1)}{(x+2)(x-2)} \div \frac{x-5}{(x+2)(x+1)} = \frac{(x-3)(x+1)}{(x-1)} \quad x \neq \pm 1, \pm 2, 5$$

$$* c) \frac{1-x^2}{1+y} \times \frac{1-y^2}{x+x^2} \div \frac{y^3-y}{x^2} = \frac{(1+x)(1-x)}{(1+y)} \times \frac{(1+y)(1-y)}{x(1+x)} \div \frac{y(y^2-1)}{x^2} = \frac{(1-y)(1-x)}{x} \times \frac{x^2}{y(y+1)(y-1)} = \frac{-x(1-x)(1-x)}{y(y+1)(y-1)}$$

$$d) \frac{x^2-y^2}{4x^2-y^2} \times \frac{4x^2+8xy+3y^2}{x+y} \div \frac{2x+3y}{2x-y} = \frac{(x-y)(x+y)}{(2x+y)(2x-y)} \times \frac{(2x+3y)(2x+y)}{(x+y)} \times \frac{(2x-y)}{(2x+3y)} = x-y$$

$x \neq \pm \frac{1}{2}y, -y, -\frac{3}{2}y$
 ↑
 Text error: restrictions in terms of y, not x

$$= \frac{-x(1-x)}{y(y+1)} \quad x \neq 0, -1 \quad y \neq -1, 0, 1$$

Text error

Text leaves -ve sign in denom

$$14.a) \frac{4}{5x} - \frac{2}{3x}$$

$$= \frac{12-10}{15x}$$

$$= \frac{2}{15x}$$

$$x \neq 0$$

$$b) \frac{5}{x+1} - \frac{2}{x-1}$$

$$= \frac{5x-5-2x-2}{(x+1)(x-1)}$$

$$= \frac{3x-7}{(x+1)(x-1)}$$

$$x \neq \pm 1$$

$$c) \frac{1}{x^2+3x-4} + \frac{1}{x^2+x-12}$$

$$= \frac{1}{(x+4)(x-1)} + \frac{1}{(x+4)(x-3)}$$

$$= \frac{x-3 + x-1}{(x+4)(x-1)(x-3)}$$

$$= \frac{2x-4}{(x+4)(x-1)(x-3)}$$

$$= \frac{2(x-2)}{(x+4)(x-1)(x-3)}$$

$$x \neq -4, 1, 3$$

$$d) \frac{1}{x^2-5x+6} - \frac{1}{x^2-9}$$

$$= \frac{1}{(x-3)(x-2)} - \frac{1}{(x-3)(x+3)}$$

$$= \frac{\cancel{x+3} - (x-2)}{(x-3)(x-2)(x+3)}$$

$$= \frac{5}{(x-3)(x-2)(x+3)}$$

$$x \neq \pm 3, 2$$

$$15.a) \frac{1}{2x} - \frac{7}{3x^2} + \frac{4}{x^3}$$

$$= \frac{3x^2 - 14x + 24}{6x^3}$$

$$x \neq 0$$

$$b) \frac{3x}{x+2} + \frac{4x}{x-6}$$

$$= \frac{3x(x-6) + 4x(x+2)}{(x+2)(x-6)}$$

$$= \frac{3x^2 - 18x + 4x^2 + 8x}{(x+2)(x-6)}$$

$$= \frac{7x^2 - 10x}{(x+2)(x-6)}$$

$$x \neq -2, 6$$

$$c) \frac{6x}{x^2-5x+6} - \frac{3x}{x^2+x-12}$$

$$= \frac{6x}{(x-3)(x-2)} - \frac{3x}{(x+4)(x-3)}$$

$$= \frac{6x(x+4) - 3x(x-2)}{(x-3)(x-2)(x+4)}$$

$$= \frac{6x^2 + 24x - 3x^2 + 6x}{(x-3)(x-2)(x+4)}$$

$$= \frac{3x^2 + 30x}{(x-3)(x-2)(x+4)} \quad x \neq -4, 2, 3$$

$$d) \frac{2(x-2)^2}{x^2+6x+5} \times \frac{3x+15}{(2-x)^2}$$

$$= \frac{2(x-2)^2}{(x+5)(x+1)} \times \frac{3(\cancel{x+5})}{(2-x)(2-x)}$$

$$= \frac{2(x-2)^2}{(x+1)} \times \frac{3}{(-1)(x-2)(-1)(x-2)}$$

$$= \frac{2(\cancel{x-2})^2}{(x+1)} \times \frac{3}{(\cancel{x-2})^2}$$

$$= \frac{6}{x+1}$$

$$x \neq -5, -1, 2$$

$$e) \frac{(x-2y)^2}{x^2-y^2} \div \frac{(x-2y)(x+3y)}{(x+y)^2}$$

$$= \frac{(x-2y)^{\cancel{2}}}{(\cancel{x+y})(x-y)} \times \frac{(x+y)^{\cancel{2}}}{(\cancel{x-2y})(x+3y)}$$

$$= \frac{(x-2y)(x+y)}{(x-y)(x+3y)}$$

$$x \neq \pm y, 2y, -3y$$

$$f) \frac{2b-5}{b^2-2b+15} + \frac{3b}{b^2+b-30} \times \frac{b^2+8b+12}{b+3}$$

$$= \frac{2b-5}{(b-5)(b+3)} + \frac{3b}{(b+6)(b-5)} \times \frac{(b+6)(b+2)}{(b+3)}$$

$$= \frac{2b-5}{(b-5)(b+3)} + \frac{3b(b+2)}{(b-5)(b+3)}$$

$$= \frac{2b-5 + 3b^2 + 6b}{(b-5)(b+3)}$$

$$= \frac{3b^2 + 8b - 5}{(b-5)(b+3)}$$

$$b \neq -6, -3, 5$$

p. 134 5-7

$$\begin{aligned} 5.a) \quad & 3m(m-1) + 2m(1-m) \\ & = 3m(m-1) - 2m(m-1) \\ & = (3m-2m)(m-1) \\ & = m(m-1) \end{aligned}$$

$$\begin{aligned} b) \quad & x^2 - 27x + 72 \\ & = (x-3)(x-24) \end{aligned}$$

$$\begin{aligned} c) \quad & 15x^2 - 7xy - 2y^2 \\ & = (5x+y)(3x-2y) \end{aligned}$$

$$\begin{array}{r|l} 1 & 3 & | & 2 \\ 15 & 5 & | & 2 \\ \hline & & & \text{Diff of 7} \end{array}$$

$$\begin{aligned} d) \quad & (2x-y+1)^2 - (x-y-2)^2 \\ & = [2x-y+1 - (x-y-2)][2x-y+1 + x-y-2] \\ & = (x+3)(3x-2y-1) \end{aligned}$$

$$\begin{aligned} e) \quad & 5xy - 10x - 3y + 6 \\ & = 5x(y-2) - 3(y-2) \\ & = (5x-3)(y-2) \end{aligned}$$

$$\begin{aligned} f) \quad & p^2 - m^2 + 6m - 9 \\ & = p^2 - (m^2 - 6m + 9) \\ & = p^2 - (m-3)^2 \\ & = (p - (m-3))(p + m - 3) \\ & = (-m + p + 3)(m + p - 3) \\ & = -(m - p - 3)(m + p - 3) \end{aligned}$$

6. x-int, y=0

$$0 = x^3 - 4x^2 - x + 4$$

$$0 = x^2(x-4) - (x-4)$$

$$0 = (x^2-1)(x-4)$$

$$0 = (x+1)(x-1)(x-4)$$

$$\therefore x\text{-int} = \pm 1, 4$$

$$\begin{array}{r|l} 1 & 2 & 3 & 6 & | & 1 & 2 \\ 6 & 3 & 2 & 1 & | & 6 & 3 \\ \hline & & & & & \text{sum of 13} \end{array}$$

$$7.a) \quad \frac{4a^2b}{5ab^2} \div \frac{6a^2b}{35ab}$$

$$= \frac{24a}{15b^2} \times \frac{7}{6a}$$

$$= \frac{14}{3b} \quad a, b \neq 0$$

$$b) \quad \frac{x-2}{x^2-x-12} \times \frac{2x-8}{x^2-4x+4}$$

$$= \frac{x-2}{(x-4)(x+3)} \times \frac{2(x-4)}{(x-2)^2}$$

$$= \frac{2}{(x+3)(x-2)}$$

$$x \neq -3, 2, 4$$

$$c) \quad \frac{5}{t^2-7t+18} + \frac{6}{t+2}$$

$$= \frac{5}{(t-9)(t+2)} + \frac{6}{t+2}$$

$$= \frac{5+6t-54}{(t-9)(t+2)}$$

$$= \frac{6t-49}{(t-9)(t+2)}$$

$$t \neq 9, -2$$

$$d) \quad \frac{4x}{6x^2+13x+6} - \frac{3x}{4x^2-9}$$

$$= \frac{4x}{(3x+2)(2x+3)} - \frac{3x}{(2x+3)(x-3)}$$

$$= \frac{4x(2x-3) - 3x(3x+2)}{(3x+2)(2x+3)(2x-3)}$$

$$= \frac{8x^2 - 12x - 9x^2 - 6x}{(3x+2)(2x+3)(2x-3)}$$

$$= \frac{-x^2 - 18x}{(3x+2)(2x+3)(2x-3)}$$

$$x \neq -\frac{2}{3}, \pm \frac{3}{2}$$

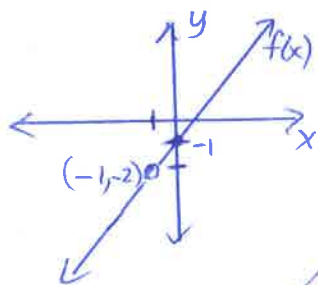
Back to Unit 2 Review Handout

$$\begin{aligned}
 4. a) f(x) &= \frac{x^3 + x^2 - x - 1}{x^2 + 2x + 1} \\
 &= \frac{x^2(x+1) - (x+1)}{(x+1)^2} \\
 &= \frac{(x^2-1)(\cancel{x+1})}{(x+1)^2} \\
 &= \frac{(x^2-1)}{(x+1)} \\
 &= \frac{(x+1)(x-1)}{(x+1)} \\
 &= x-1
 \end{aligned}$$

Hole: sub $x = -1$

$$f(-1) = -1 - 1 = -2$$

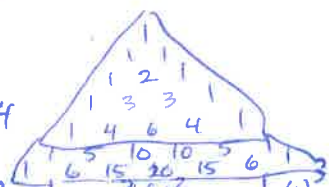
\therefore There is a hole at $(-1, -2)$



$$\begin{aligned}
 D: \{x \in \mathbb{R} \mid x \neq -1\} \\
 R: \{y \in \mathbb{R} \mid y \neq -2\}
 \end{aligned}$$

5. p. 469

$$\begin{aligned}
 23. a) (a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \\
 &= a^4 + 24a^3 + 216a^2 + 864a + 1296
 \end{aligned}$$



$$\begin{aligned}
 b) (b-3)^5 &= b^5 + 5b^4(-3) + 10b^3(-3)^2 + 10b^2(-3)^3 + 5b(-3)^4 + (-3)^5 \\
 &= b^5 - 15b^4 + 90b^3 - 270b^2 + 405b - 243
 \end{aligned}$$

$$\begin{aligned}
 c) (2c+5)^3 &= (2c)^3 + 3(2c)^2(5) + 3(2c)(5)^2 + (5)^3 \\
 &= 8c^3 + 60c^2 + 150c + 125
 \end{aligned}$$

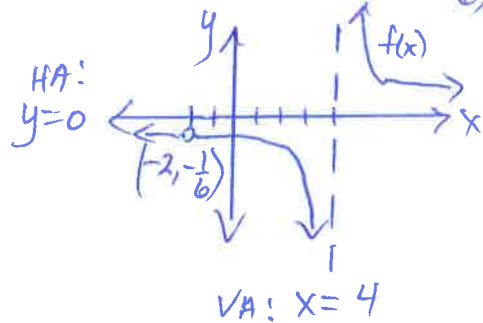
$$\begin{aligned}
 b) f(x) &= \frac{x+2}{x^2-2x-8} \\
 &= \frac{(x+2)}{(x+2)(x-4)} \\
 &= \frac{1}{x-4}
 \end{aligned}$$

HA: $y = 0$

VA: $x = 4$

Hole: $f(-2) = \frac{1}{-2-4} = -\frac{1}{6}$

\therefore Hole at $(-2, -\frac{1}{6})$



$$\begin{aligned}
 D: \{x \in \mathbb{R} \mid x \neq -2, 4\} \\
 R: \{y \in \mathbb{R} \mid y \neq 0, -\frac{1}{6}\}
 \end{aligned}$$

$$\begin{aligned}
 d) (4-3d)^6 &= (4)^6 + 6(4)^5(-3d) + 15(4)^4(-3d)^2 + 20(4)^3(-3d)^3 \\
 &\quad + 15(4)^2(-3d)^4 + 6(4)(-3d)^5 + (-3d)^6 \\
 &= 4096 - 18432d + 34560d^2 - 34560d^3 \\
 &\quad + 19440d^4 - 5832d^5 + 729d^6
 \end{aligned}$$

$$\begin{aligned}
 e) (5e-2f)^4 &= (5e)^4 + 4(5e)^3(-2f) + 6(5e)^2(-2f)^2 + 4(5e)(-2f)^3 + (-2f)^4 \\
 &= 625e^4 - 1000e^3f + 600e^2f^2 - 160ef^3 + 16f^4
 \end{aligned}$$