

U4/L5 Solving Exponential Equ^{ns}

Goal: Use algebra and or exponent laws to get to the point when you have only one power term, with the same base, on each side of the equal sign. Then, the exponents will be equal and you can solve for the variable.

Ex. solve

a) $3^x = 81$

$$3^x = 3^4$$

$$\therefore x = 4$$

b) $16^{2y} = 8^{y+1}$

$$(2^4)^{2y} = (2^3)^{y+1}$$

$$2^{8y} = 2^{3y+3}$$

$$\therefore 8y = 3y + 3$$

$$5y = 3$$

$$y = \frac{3}{5}$$

c) $8^{2x} = \frac{1}{16^{x-1}}$

$$(2^3)^{2x} = (16^{x-1})^{-1}$$

$$2^{6x} = 16^{-x+1}$$

$$2^{6x} = (2^4)^{-x+1}$$

$$2^{6x} = 2^{-4x+4}$$

$$\therefore 6x = -4x + 4$$

$$10x = 4$$

$$x = \frac{2}{5}$$

d) $32^{x+3} = 16^{-3x}$

$$(2^5)^{x+3} = (2^4)^{-3x}$$

$$2^{5x+15} = 2^{-12x}$$

$$\therefore 5x + 15 = -12x$$

$$17x = -15$$

$$x = \frac{-15}{17}$$

e) Algebra Required!

$$2(3^{y-2}) = 18$$

← Divide b.s. by 2

$$3^{y-2} = 9$$

$$3^{y-2} = 3^2$$

$$\therefore y - 2 = 2$$

$$y = 4$$

→

f) Factoring Required!

$$10^{n+4} + 10^{n+3} = 11$$

$$10^{n+3} [10^1 + 1] = 11$$

$$10^{n+3} (11) = 11$$

$$10^{n+3} = 1$$

$$10^{n+3} = 10^0$$

$$\therefore n+3=0$$

$$n = -3$$

Practise Q's Handout

1. a) $2^x = 16$

$$2^x = 2^4$$

$$\therefore x = 4$$

f) $729 = 9^x$

$$3^6 = 3^{2x}$$

$$\therefore 2x = 6$$

$$x = 3$$

g) $(-3)^x = -27$

$$(-3)^x = (-3)^3$$

$$\therefore x = 3$$

h) $(-2)^x = -32$

$$(-2)^x = (-2)^5$$

$$\therefore x = 5$$

k) $-2^x = -16$

$$2^x = 16$$

$$2^x = 2^4$$

$$\therefore x = 4$$

l) $-4^y = -64$

$$4^y = 64$$

$$4^y = 4^3$$

$$\therefore y = 3$$

m) $-5^x = -625$

$$5^x = 625$$

$$5^x = 5^4$$

$$\therefore x = 4$$

n) $(-1)^x = 1$

$$(-1)^x = (-1)^0$$

$$\therefore x = 0$$

and any
even integer

o) $(-1)^m = -1$

$$(-1)^m = (-1)^1$$

$$\therefore m = 1$$

or any
odd integer

2. a) $7^{w-2} = 49$

$$7^{w-2} = 7^2$$

$$\therefore w-2 = 2$$

$$w = 4$$

c) $2^{1-x} = 128$

$$2^{1-x} = 2^7$$

$$\therefore 1-x = 7$$

$$x = -6$$

e) $5^{3x-1} = 25$

$$5^{3x-1} = 5^2$$

$$\therefore 3x-1 = 2$$

$$3x = 3$$

$$x = 1$$

f) $-81 = -3^{2x+8}$

$$81 = 3^{2x+8}$$

$$3^4 = 3^{2x+8}$$

$$\therefore 4 = 2x+8$$

$$-4 = 2x$$

$$x = -2$$

h) $3^{2-2x} = 1$

$$3^{2-2x} = 3^0$$

$$\therefore 2-2x = 0$$

$$-2x = -2$$

$$x = 1$$

3. a) $6^{x+3} = 6^{2x}$

$$\therefore x+3 = 2x$$

$$x = 3$$

d) $2^{4x-7} = 2^{2x+1}$

$$\therefore 4x-7 = 2x+1$$

$$2x = 8$$

$$x = 4$$

$$4. a) 16^{2x} = 8^{3x}$$

$$(2^4)^{2x} = (2^3)^{3x}$$

$$2^{8x} = 2^{9x}$$

$$\therefore 8x = 9x$$

$$\boxed{x = 0}$$

$$5. a) 2^{x+5} = 4^{x+2}$$

$$2^{x+5} = 2^{2x+4}$$

$$\therefore x+5 = 2x+4$$

$$\boxed{x = 1}$$

$$6. a) 5^{4-x} = \frac{1}{5}$$

$$5^{4-x} = 5^{-1}$$

$$\therefore 4-x = -1$$

$$\boxed{x = 5}$$

$$7. d) 9^{-x} = 3$$

$$3^{-2x} = 3^1$$

$$\therefore -2x = 1$$

$$\boxed{x = -\frac{1}{2}}$$

$$8. a) 9^{x+1} = 27^{2x}$$

$$3^{2x+2} = 3^{6x}$$

$$\therefore 2x+2 = 6x$$

$$4x = 2$$

$$\boxed{x = \frac{1}{2}}$$

$$e) 16^{2p+1} = 8^{3p+1}$$

$$(2^4)^{2p+1} = (2^3)^{3p+1}$$

$$2^{8p+4} = 2^{9p+3}$$

$$\therefore 8p+4 = 9p+3$$

$$\boxed{p = 1}$$

$$c) 9^{2q-6} = 3^{q+6}$$

$$3^{4q-12} = 3^{q+6}$$

$$\therefore 4q-12 = q+6$$

$$3q = 18$$

$$\boxed{q = 6}$$

$$b) 10^{y-2} = \frac{1}{10000}$$

$$10^{y-2} = 10^{-4}$$

$$\therefore y-2 = -4$$

$$\boxed{y = -2}$$

$$e) 2^{9x} = \frac{1}{8}$$

$$2^{9x} = 2^{-3}$$

$$\therefore 9x = -3$$

$$\boxed{x = -\frac{1}{3}}$$

$$f) 16^{3+k} = 32^{1-2k}$$

$$2^{12+4k} = 2^{5-10k}$$

$$12+4k = 5-10k$$

$$14k = -7$$

$$\boxed{k = -\frac{1}{2}}$$

$$f) (-8)^{1-2x} = (-32)^{1-x}$$

$$(-2)^{3-6x} = (-2)^{5-5x}$$

$$\therefore 3-6x = 5-5x$$

$$\boxed{x = -2}$$

$$e) 27^{y-1} = 9^{2y-4}$$

$$3^{3y-3} = 3^{4y-8}$$

$$\therefore 3y-3 = 4y-8$$

$$\boxed{y = 5}$$

$$f) \frac{1}{256} = 2^{2-5w}$$

$$2^{-8} = 2^{2-5w}$$

$$\therefore -8 = 2-5w$$

$$5w = 10$$

$$\boxed{w = 2}$$

$$i) 9^{2x+1} = 27$$

$$3^{4x+2} = 3^3$$

$$\therefore 4x+2 = 3$$

$$4x = 1$$

$$\boxed{x = \frac{1}{4}}$$

$$9. a) 5 = 25^{\frac{x}{2}}$$

$$5^1 = 5^x$$

$$\therefore \boxed{x=1}$$

$$c) 9^{\frac{y}{5}} = 27$$

$$3^{\frac{2y}{5}} = 3^3$$

$$\therefore \frac{2y}{5} = 3$$

$$2y = 15$$

$$\boxed{y = \frac{15}{2}}$$

$$f) \left(\frac{3}{2}\right)^{\frac{m}{2}} = \frac{4}{9}$$

$$\left(\frac{3}{2}\right)^{\frac{m}{2}} = \left(\frac{3}{2}\right)^{-2}$$

$$\therefore \frac{m}{2} = -2$$

$$\boxed{m = -4}$$

$$10. a) 3(5^{x+1}) = 15$$

$$5^{x+1} = 5^1$$

$$\therefore x+1 = 1$$

$$\boxed{x=0}$$

$$c) 5(4^x) = 10$$

$$4^x = 2$$

$$2^{2x} = 2^1$$

$$\therefore 2x = 1$$

$$\boxed{x = \frac{1}{2}}$$

$$f) 27(3^{3x+1}) = 3$$

$$3^{3x+1} = \frac{1}{9}$$

$$3^{3x+1} = 3^{-2}$$

$$\therefore 3x+1 = -2$$

$$3x = -3$$

$$\boxed{x = -1}$$

$$11. a) 2^{x+2} - 2^x = 48$$

$$2^x(2^2 - 1) = 48$$

$$2^x(3) = 48$$

$$2^x = 16$$

$$2^x = 2^4$$

$$\therefore \boxed{x=4}$$

$$c) 2^{a+5} + 2^a = 1056$$

$$2^a(2^5 + 1) = 1056$$

$$2^a(33) = 1056$$

$$2^a = 32$$

$$2^a = 2^5$$

$$\therefore \boxed{a=5}$$

$$e) 3^{x+3} - 3^{x+1} = 648$$

$$3^{x+1}(3^2 - 1) = 648$$

$$3^{x+1}(8) = 648$$

$$3^{x+1} = 81$$

$$3^{x+1} = 3^4$$

$$\therefore x+1 = 4$$

$$\boxed{x=3}$$

$$g) 2^{x+2} - 2^{x+5} = -7$$

$$2^{x+2}(1 - 2^3) = -7$$

$$2^{x+2}(-7) = -7$$

$$2^{x+2} = 1$$

$$2^{x+2} = 2^0$$

$$\therefore x+2 = 0$$

$$\boxed{x = -2}$$

$$i) 5^{n+2} - 5^{n+3} = -2500$$

$$5^{n+2}(1 - 5^1) = -2500$$

$$5^{n+2}(-4) = -2500$$

$$5^{n+2} = 625$$

$$5^{n+2} = 5^4$$

$$\therefore n+2 = 4$$

$$\boxed{n = 2}$$

$$16. \quad S = 0.8^d \times 100\%$$

$$a) \quad \text{set } S = 64$$

$$64 = 0.8^d \times 100$$

$$0.64 = 0.8^d$$

$$(0.8)^2 = 0.8^d$$

$\therefore d = 2\text{m}$, the scuba diver's depth is 2m if 64% sunlight reaches the diver.

$$b) \quad \text{sub } d = 10$$

$$S = 0.8^{10} \times 100$$

$$\approx 11\%$$

$\therefore$ Only 11% sunlight reaches the diver at a depth of 10m

$$19. a) \quad \frac{27^x}{9^{2x-1}} = 3^{x+4}$$

$$\frac{3^{3x}}{3^{4x-2}} = 3^{x+4}$$

$$3^{-x+2} = 3^{x+4}$$

$$\therefore -x+2 = x+4$$

$$-2 = 2x$$

$$\boxed{x = -1}$$

$$b) \quad 27^x (9^{2x-1}) = 3^{x+4}$$

$$3^{3x} (3^{4x-2}) = 3^{x+4}$$

$$3^{7x-2} = 3^{x+4}$$

$$\therefore 7x-2 = x+4$$

$$6x = 6$$

$$\boxed{x = 1}$$

$$c) \quad 27^{x+1} = \left(\frac{1}{9}\right)^{2x-5}$$

$$3^{3x+3} = (3^{-2})^{2x-5}$$

$$3^{3x+3} = 3^{-4x+10}$$

$$\therefore 3x+3 = -4x+10$$

$$\boxed{x = 1}$$

$$22. a) \quad \frac{2^{2x+1}}{2^{x-3}} = 4$$

$$2^{x+4} = 2^2$$

$$\therefore x+4 = 2$$

$$x = -2$$

$$c) \quad \frac{8^{x+2}}{4^{x+3}} = 16^{x-3}$$

$$\frac{2^{3x+6}}{2^{2x+6}} = 2^{4x-12}$$

$$2^x = 2^{4x-12}$$

$$\therefore x = 4x-12$$

$$3x = 12$$

$$\boxed{x = 4}$$

$$20. a) \quad 2^{x^2+2x} = 2^{x+6}$$

$$\therefore x^2+2x = x+6$$

$$x^2+x-6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3, 2$$

$$20. c) \quad 2^{2x^2-3x} = 2^{x^2-2x+12}$$

$$\therefore 2x^2-3x = x^2-2x+12$$

$$x^2-x-12 = 0$$

$$(x-4)(x+3) = 0$$

$$\therefore x = 4, -3$$

Practise

A

1. Solve.

a) $2^x = 16$

c) $2^x = 128$

e) $4^y = 256$

g) $(-3)^x = -27$

i) $(-5)^x = 25$

k) $-2^x = -16$

m) $-5^x = -625$

o) $(-1)^m = -1$

b) $3^x = 27$

d) $5^x = 125$

f) $729 = 9^x$

h) $(-2)^x = -32$

j) $81 = (-3)^x$

l) $-4^y = -64$

n) $(-1)^x = 1$

2. Solve.

a) $7^{w-2} = 49$

c) $2^{1-x} = 128$

e) $5^{3x-1} = 25$

g) $4^{x-1} = 1$

i) $(-1)^{2x} = 1$

b) $3^{x+4} = 27$

d) $4^{3k} = 64$

f) $-81 = -3^{2x+8}$

h) $3^{2-2x} = 1$

3. Solve and check.

a) $6^{x+3} = 6^{2x}$

c) $3^{2y+3} = 3^{y+5}$

b) $2^{x+3} = 2^{2x-1}$

d) $2^{4x-7} = 2^{2x+1}$

4. Solve.

a) $16^{2x} = 8^{3x}$

c) $27^{x-1} = 9^{2x}$

e) $16^{2p+1} = 8^{3p+1}$

b) $4^r = 8^{r+1}$

d) $25^{2-c} = 125^{2c-4}$

f) $(-8)^{1-2x} = (-32)^{1-x}$

5. Solve and check.

a) $2^{x+5} = 4^{x+2}$

c) $9^{2q-6} = 3^{q+6}$

e) $27^{y-1} = 9^{2y-4}$

b) $2^x = 4^{x-1}$

d) $4^x = 8^{x+1}$

f) $8^{x+3} = 16^{2x+1}$

6. Solve and check.

a) $5^{4-x} = \frac{1}{5}$

c) $6^{3x-7} = \frac{1}{6}$

e) $5^{2n+1} = \frac{1}{125}$

b) $10^{y-2} = \frac{1}{10\,000}$

d) $3^{3x-1} = \frac{1}{81}$

f) $\frac{1}{256} = 2^{2-5w}$

7. Solve and check.

a) $4^x = 8$

c) $(-8)^y = -2$

e) $2^{2x} = \frac{1}{8}$

g) $2^x = 16^4$

i) $9^{2x+1} = 27$

b) $64^z = 16$

d) $9^{-x} = 3$

f) $9^{6x} = \frac{1}{27}$

h) $2^{-2x} = 32$

8. Solve and check.

a) $9^{x+1} = 27^{2x}$

c) $36^{t-2} = 216^{-2t}$

e) $25^{1-3x} = 125^{-x}$

b) $16^y = 64^{2y-1}$

d) $8^{2x-1} = 16^{x-1}$

f) $16^{3+k} = 32^{1-2k}$

9. Solve and check.

a) $5 = 25^{\frac{x}{2}}$

c) $9^{\frac{y}{2}} = 27$

e) $4^{\frac{z}{4}} = \frac{1}{8}$

b) $8 = 2^{\frac{x}{3}}$

d) $\frac{1}{2} = 2^{\frac{a}{3}}$

f) $\left(\frac{3}{2}\right)^{\frac{m}{2}} = \frac{4}{9}$

10. Solve.

a) $3(5^{x+1}) = 15$

b) $2(3^{y-2}) = 18$

c) $5(4^z) = 10$

d) $2(4^{u+1}) = 8$

e) $2 = 6(3^{4f-2})$

f) $27(3^{3x+1}) = 3$

11. Solve and check.

a) $2^{x+2} - 2^x = 48$

b) $4^{x+3} + 4^x = 260$

c) $2^{a+5} + 2^a = 1056$

d) $6^{x+1} + 6^{x+2} = 7$

e) $3^{x+3} - 3^{x+1} = 648$

f) $10^{x+4} + 10^{x+3} = 11$

g) $2^{x+2} - 2^{x+5} = -7$

h) $3^{m+1} + 3^{m+2} - 972 = 0$

i) $5^{n+2} - 5^{n+3} = -2500$

16. **Scuba diving** The percent of sunlight, s , that reaches a scuba diver under water can be modelled by the equation

$$s = 0.8^d \times 100\%$$

where d is the depth of the diver, in metres.

- a) At what depth does 64% of sunlight reach the diver?
 b) What percent of sunlight reaches the diver at a depth of 10 m, to the nearest percent?

19. Solve.

a) $\frac{27^x}{9^{2x-1}} = 3^{x+4}$

b) $27^x(9^{2x-1}) = 3^{x+4}$

c) $27^{x+1} = \left(\frac{1}{9}\right)^{2x-5}$

20. Solve.

a) $2^{x^2+2x} = 2^{x+6}$

b) $3^{x^2-2x} = 3^{x-2}$

c) $2^{2x^2-3x} = 2^{x^2-2x+12}$

22. Solve and check.

a) $\frac{2^{2x+1}}{2^{x-3}} = 4$

b) $\frac{9^{x+4}}{27^{x-1}} = 81$

c) $\frac{8^{x+2}}{4^{x+3}} = 16^{x-3}$

ANSWERS

1. a) 4 b) 3 c) 7 d) 3 e) 4 f) 3 g) 3 h) 5 i) 2 j) 4 k) 4 l) 3 m) 4
 n) x any even integer o) m any odd integer 2. a) 4 b) -1 c) -1
 d) 1 e) 1 f) -2 g) 1 h) 1 i) all values of x 3. a) 3 b) 4 c) 2 d) 4
 e) 2 f) -2 4. a) 0 b) -3 c) -3 d) 2 e) 1 f) -2 5. a) 1 b) 2 c) 6
 d) -3 e) 5 f) 1 6. a) 5 b) -2 c) 2 d) -1 e) -2 f) 2 7. a) $\frac{3}{2}$
 b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $-\frac{1}{2}$ e) $-\frac{1}{3}$ f) $-\frac{1}{4}$ g) 16 h) $-\frac{5}{2}$ i) $\frac{1}{4}$ 8. a) $\frac{1}{2}$ b) $\frac{3}{4}$
 c) $\frac{1}{2}$ d) $-\frac{1}{2}$ e) $\frac{2}{3}$ f) $-\frac{1}{2}$ 9. a) 1 b) 9 c) $\frac{15}{2}$ d) -3 e) -6 f) -4
 10. a) 0 b) 4 c) $\frac{1}{2}$ d) $-\frac{3}{2}$ e) $\frac{1}{4}$ f) -1 11. a) 4 b) 1 c) 5 d) -1 e) 3
 f) -3 g) -2 h) 4 i) 2

16. a) 2 m

- b) 11% 17. a) 5 h b) 20.4 years c) 30 s 18. 59.6 h 19. a) -1
 b) 1 c) 1 20. a) 2, -3 b) 1, 2 c) 4, -3 21. 16 days 22. a) -2
 b) 7 c) 4 23. $x = -17$, $y = 2$