U4/L9

Properties of Exponential Functions & Their Graphs

We already know that an exponential function has a variable as part of an exponent. In this activity we will examine the properties observed in the table of values and in the graphs of exponential functions.

Problem #1 (reviewing differences)

a) Use differences to show that the table below represents a linear relationship. (see page 141 of your textbook for a review of first and second differences)

x	y
-1	9
0	11
1	13
2	15
3	17

1st Diff
$11-9=2$
$13-11=2$
2
2

$\because$ 1st diff is a constant
$\therefore$ the relationship is linear

b) Use differences to show that the table below represents a quadratic relationship.

x	y
1	3
2	2
3	3
4	6
5	11
6	18

1st Diff	2nd Diff
$2-3=-1$	$1-(-1)=2$
$3-2=1$	$3-1=2$
$6-3=3$	$5-3=2$
5	$7-5=2$
7	

$\because$ the 2nd diff is a constant
$\therefore$ the rel. is quadratic

Problem #2

a) Calculate the first and second differences for the table below. What do you notice about the first and second differences?

x	y
1	4
2	8
3	16
4	32
5	64

1st Diff	2nd Diff
4	4
8	8
16	16
32	

b) Using the same table of values show that there is a "constant ratio". The term *constant ratio* is also called a *growth factor* or *decay factor.* It is calculated similar to first differences, only with division instead of subtraction.

First Quotient (divide)

x	y
1	4
2	8
3	16
4	32
5	64

FQ
$8\div4=2$
$16\div8=2$
$\frac{32}{16}=2$
$\frac{64}{32}=2$

$\because$ the FQ is a constant
$\therefore$ the relationship is exponential

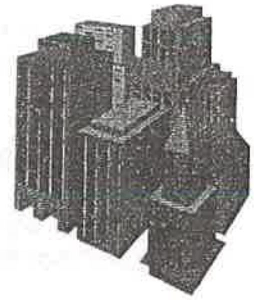
c) What is the equation for the above table of values?

$$y=2(2)^x$$

starting amt when $x=0$

constant ratio/FQ value

Problem #3 Examining Population Growth



a) The population of a city is shown below. Is the population growth linear, quadratic or exponential?

Year	Population
2000	185,000
2001	200,000
2002	215,000
2003	230,000

1st Diff
15 000
15 000
15 000

∵ 1st Diff is constant
∴ linear

b) The population of a second city is shown below. Is the population growth linear, quadratic or exponential?

Year	Population
2000	95,000
2001	97,850
2002	100,786
2003	103,810

FQ
1.03
1.03
1.03

∵ FQ is constant
∴ exponential

c) The population of a third city is shown below. Is the population growth linear, quadratic or exponential?

Year	Population
2000	110,000
2001	111,400
2002	115,600
2003	122,600
2004	132,400

1st Diff	2nd Diff
1400	2800
4200	2800
7000	2800
9800	

∵ 2nd Diff is constant
∴ quadratic

d) Find an equation that models the growth of the first and second cities.

a) $m = 15000$ ← 1st Diff value and $b = 185\,000$ (starting amt.)

∴ $y = 15000x + 185\,000$ ← test $(3, 230\,000)$ ✓

b) base/growth factor is 1.03 (FQ value) and starting amt is 95 000

∴ $y = 95000(1.03)^x$ where x is # years since the yr. 2000

test $(2, 100\,786)$ ✓

Problem #4: Examing the graph of the function $f(x) = b^x$

In the first unit of this course we look at parent functions, and families of functions. The parent function for all exponential functions is $f(x) = b^x$. (There is more than one parent for different values of b).

Each function below is in the form $f(x) = b^x$ with different values for b. **Complete each table of values below.** Leave values as rational numbers. **Graph each function on graph paper.**

a) $f_1(x) = 2^x$

x	$f(x)$
-4	1/16
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8
4	16

b) $f_2(x) = 5^x$

x	$f_2(x)$
-4	1/625
-3	1/125
-2	1/25
-1	1/5
0	1
1	5
2	25
3	125
4	625

c) $f_3(x) = 10^x$

x	$f_3(x)$
-4	1/10 000
-3	1/1000
-2	1/100
-1	1/10
0	1
1	10
2	100
3	1000
4	10 000

d) $f_4(x) = \left(\frac{1}{2}\right)^x$

x	$f_4(x)$
-4	16
-3	8
-2	4
-1	2
0	1
1	1/2
2	1/4
3	1/8
4	1/16

e) $f_5(x) = \left(\frac{1}{5}\right)^x$

x	$f_5(x)$
-4	625
-3	125
-2	25
-1	5
0	1
1	1/5
2	1/25
3	1/125
4	1/625

f) $f_6(x) = \left(\frac{1}{10}\right)^x$

x	$f_6(x)$
-4	10 000
-3	1000
-2	100
-1	10
0	1
1	1/10
2	1/100
3	1/1000
4	1/10 000

Examine each of the 6 graphs you made and complete the table below.

Function	Constant Ratio	Increasing or Decreasing ?	y-intercept	Equation of Asymptote	Domain	Range
$f(x)=2^x$	$\times 2$	incr.	(0,1)	$y=0$	$\{x \in R\}$	$\{y \in R \mid y \neq 0\}$
$f(x)=5^x$	$\times 5$	incr.	(0,1)	$y=0$	$\{x \in R\}$	$\{y \in R \mid y \neq 0\}$
$f(x)=10^x$	$\times 10$	incr.	(0,1)	$y=0$	$\{x \in R\}$	$\{y \in R \mid y \neq 0\}$
$f(x)=\left(\frac{1}{2}\right)^x$	$\times \frac{1}{2}$ (or $\div 2$)	decr.	(0,1)	$y=0$	$\{x \in R\}$	$\{y \in R \mid y \neq 0\}$
$f(x)=\left(\frac{1}{5}\right)^x$	$\times \frac{1}{5}$ (or $\div 5$)	decr.	(0,1)	$y=0$	$\{x \in R\}$	$\{y \in R \mid y \neq 0\}$
$f(x)=\left(\frac{1}{10}\right)^x$	$\times \frac{1}{10}$ (or $\div 10$)	decr.	(0,1)	$y=0$	$\{x \in R\}$	$\{y \in R \mid y \neq 0\}$

Problem #5

Complete the following:

In general the graph of $f(x)=b^x$ has the following properties:

a) It has a y-intercept at: (0,1)

b) It has an asymptote at: $y=0$

c) If b > 1 the graph is: increasing

d) If 0 < b < 1 the graph is: decreasing

e) It has a domain of: $\{x \in R\}$

f) It has a range of: $\{y \in R \mid y \neq 0\}$

Properties of Exponential Functions

1. Examine each table below. Determine if the relationship is linear, exponential. Then find the equation that represents each table.

(a) Exponential $y=3^x$

x	y	FQ
0	1	3
1	3	3
2	9	3
3	27	

(b) Linear $y=3x-1$

x	y	1^{st} Diff
0	−1	3
1	2	3
2	5	3
3	8	

(c) Linear $y=5x+1$

x	y	1^{st} Diff
0	1	5
1	6	5
2	11	5
3	16	

(d)

x	y	FQ
0	1	5
1	5	5
2	25	5
3	125	5
4	625	5
5	3125	5
6	15 625	

Exponential

$y=5^x$

(e)

x	y	1^{st} Diff
0	12	−2
1	10	−2
2	8	−2
3	6	−2
4	4	−2
5	2	−2
6	0	

Linear

$y=-2x+12$

(f)

x	y
3	135
0	5
4	405
2	45
1	15
6	3645
5	1215

x	y	FQ
0	5	3
1	15	3
2	45	3
3	135	3
4	405	3
5	1215	3
6	3645	

Exponential

$y=5(3)^x$

2. Text page 243 #2 and page 267 #1

p.243 #2. a) exponential where $0<b<1$

b) exp. $b>1$

c) linear (straight line)

d) quadratic (parabola)

p.267 #1. a) If $x>1$, then $x^2>x^{-2}$, since x^2 gives a value greater than 1 but x^{-2} gives a fraction between 0 and 1

b) $x^{-2}>x^2$ when $-1<x<1$ and $x\neq 0$

eg. if $x=-\frac{1}{2}$, then $x^{-2}=\left(-\frac{1}{2}\right)^{-2} = \left(-\frac{2}{1}\right)^2$ or 4 but $x^2=\left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

Answers

a) $y=3^x$ b) $y=3x-1$ c) $y=5x+1$ d) $y=5^x$ e) $y=-2x+12$ f) $y=5(3)^x$