

Unit 4: Exponential Functions – Review

Review Problems

1. **Exponent Laws** text page 267 #2 – 5, 7 – 9 (find equation for linear/exponential tables in #9)

2. **Solving Exponential Equations.** Solve the following:

a) $3^{2y-3} = 9$ b) $2^{3x+2} = \frac{1}{16}$ c) $4(6^{x+2}) = 144$ d) $2^{x+2} - 2^{x+3} = -64$

e) $\frac{5^{x-1}}{25} = 125$

3. **Exponential Growth & Decay Problems.** text page 269: 14, 16, 17abd and page 270 #5a

4. Transformations of Exponential Functions

a) Find the x/y-intercepts for the function $f(x) = 2(3)^{-x} - 18$

b) Graph the exponential function $g(x) = -\left(\frac{1}{2}\right)^{-x+1} - 2$ using mapping notation.

c) Text page 253 #14

5. Compound Interest.

a) Find the final value of a \$6500 G.I.C. invested for 6 years at 3.4% interest compounded quarterly.

b) Suppose that Jack has a savings account with interest compounded monthly. His starting balance is \$3100. After one year the balance is \$3165.73. What is the annual interest rate for the account?

Answers

2. a) 5/2 b) -2 c) 0 d) 4 e) 6 4 a) (0, -16) and (-2, 0) 5. a) \$7964.07 b) 2.1%

U4 Review

#2 if denom should be $(-3)^4)^3$
 a-e only

1. Text. p267 2-5, 7-9 (find eqn for lin/exp fun in #9)

2. a) $(-7)^3(-7)^{-4} = (-7)^{-1} = -\frac{1}{7}$
 b) $\frac{(-2)^8}{(-2)^3} = (-2)^5 = -32$
 c) $\frac{(5)^3(5)^6}{5^3} = \frac{5^9}{5^3} = 5^6 = 15625$
 d) $\frac{4^{-10}(4^{-3})^6}{(4^4)^8} = \frac{4^{-10} \cdot 4^{-18}}{4^{-32}} = 4^{-28} = \frac{1}{4^{28}}$
 e) $(11)^9 \left(\frac{1}{11}\right)^7 = 11^2 = 121$
 f) $\frac{(-3)^7(-3)^4}{(-3^4)^3} = \frac{(-3)^{11}}{(-3)^{12}} = (-3)^{-1} = -\frac{1}{3}$

3. a) $\sqrt[3]{x^7} = x^{7/3}$ b) $y^{8/5} = \sqrt[5]{y^8}$ c) $(\sqrt{p})^n = p^{n/2}$ d) $m^{1.25} = m^{5/4} = (\sqrt[4]{m})^5$

4. a) $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$
 b) $\left(\frac{16}{225}\right)^{-1/2} = \left(\frac{225}{16}\right)^{1/2} = \frac{15}{4}$
 c) $\frac{81^{-1/4}}{\sqrt[3]{-125}} = \frac{1}{\sqrt[4]{81}} \div (-5) = \frac{1}{3} \times \frac{1}{-5} = -\frac{1}{15}$
 d) $(\sqrt{-27})^4 = (-3)^4 = 81$
 e) $(\sqrt{5-32})(\sqrt[4]{64})^5 = (-2)(2)^5 = -64$
 f) $\sqrt[6]{(-2)^3}^2 = \sqrt[6]{(-2)^6} = -2$

5. a) $a^{3/2}(a^{-3/2}) = a^0 = 1$
 b) $\frac{b^{0.8}}{b^{-0.2}} = b^{1.0} = b$
 c) $\frac{c^{5/6}}{c^2} = c^{-7/6} = \frac{1}{c^{12/6}} = \frac{1}{c^2}$
 d) $\frac{d^{-5}d^{1/2}}{(d^{-3})^2} = \frac{d^{-9/2}}{d^{-6}} = d^{3/2}$
 e) $(e^{-2})^{7/2} = e^{-7} = \frac{1}{e^7}$
 f) $\left(\left(\frac{1}{f}\right)^{1/5}\right)^{-1} = f^{1/5}$

7. a) $(5x)^2(2x)^3 = 25x^2 \cdot 8x^3 = 200x^5$
 sub $x = -2$
 $200(-2)^5 = 200(-32) = -6400$

b) $\frac{8m^{-5}}{(2m)^{-3}} = 8m^{-5} \cdot (2m)^3 = 8m^{-5} \cdot 8m^3 = 64m^{-2} = \frac{64}{m^2}$
 when $m = 4$
 $\frac{64}{4^2} = \frac{64}{16} = 4$

c) $\frac{2w(3w^{-2})}{(2w)^2} = \frac{6w^{-1}}{4w^2} = \frac{3}{2w^3}$
 sub $w = -3$
 $\frac{3}{2(-3)^3} = \frac{3}{-54} = -\frac{1}{18}$

d) $\frac{(9y)^2}{(3y^{-1})^3} = \frac{81y^2}{27y^{-3}} = 3y^5$
 sub $y = -2$
 $3(-2)^5 = 3(-32) = -96$

e) $(6(x-4)^3)^{-1} = \frac{1}{6(x-4)^3}$
 when $x = -2$
 $\frac{1}{6(-2-4)^3} = \frac{1}{6(-6)^3} = \frac{1}{6(-216)} = -\frac{1}{1296}$

$$7.f) \frac{(-2x^{-2})^3 (6x)^2}{2(-3x^{-1})^3}$$

$$= \frac{(-2)^3 x^{-6} \cdot \frac{18}{3} x^2}{12 (-3)^3 x^{-3}}$$

$$= \frac{-8 \cdot 6 \cdot x^{-4}}{-3 \cdot 27 x^{-3}}$$

$$= \frac{16}{3x}$$

when $x = \frac{1}{2}$

$$16 \div 3\left(\frac{1}{2}\right)$$

$$= 16 \times \frac{2}{3}$$

$$= \frac{32}{3}$$

$$8.a) \sqrt[3]{27x^3y^9}$$

$$= (27x^3y^9)^{\frac{1}{3}}$$

$$= 3xy^3$$

$$8.b) \sqrt{\frac{a^6b^5}{a^8b^3}}$$

$$= \sqrt{a^{-2}b^2}$$

$$= (a^{-2}b^2)^{\frac{1}{2}}$$

$$= a^{-1}b$$

$$= \frac{b}{a}$$

$$8.c) \frac{m^{\frac{3}{2}}n^{-2}}{m^{\frac{7}{2}}n^{-3/2}}$$

$$= m^{-2}n^{-1/2}$$

$$= \frac{1}{m^2n^{1/2}}$$

$$8.d) \sqrt[4]{x^{-16}(x^6)^{-6}}$$

$$= \frac{(x^6)^{-11/2}}{x^{-16}}$$

$$= (x^{-16} \cdot x^{-36})^{\frac{1}{4}}$$

$$= \frac{(x^{-52})^{\frac{1}{4}}}{x^{-22}}$$

$$= \frac{x^{-13}}{x^{-22}}$$

$$= x^9$$

$$e) ((-x^{0.5})^3)^{-1.2}$$

$$= (-1)^3 x^{-1.8}$$

$$= \frac{-1}{x^{1.8}}$$

$$8.f) \sqrt{\frac{x^6(y^3)^{-2}}{(x^3y)^{-2}}}$$

$$= \frac{(x^6y^{-6})^{\frac{1}{2}}}{x^{-6}y^{-2}}$$

$$= \frac{x^3y^{-3}}{x^{-6}y^{-2}}$$

$$= \frac{x^9}{y}$$

9. a) 2nd diff is 10
quadratic

b) 1st diff = 30
(x values 2 units apart
so $m=15$)
 $\therefore y = 15x - 45$ ✓

c) $FQ_1 = 4$
 $\therefore y = 4^x + 9$

d) $FQ_1 = \frac{1}{2}$
 $\therefore y = 10\left(\frac{1}{2}\right)^x$

e) $FQ_1 = \frac{1}{2}$
 $\therefore y = 500\left(\frac{1}{2}\right)^x$

Handout Q's

2.a) $3^{2y-3} = 9$

$$3^{2y-3} = 3^2$$

$$\therefore 2y-3 = 2$$

$$2y = 5$$

$$y = \frac{5}{2}$$

b) $2^{3x+2} = \frac{1}{16}$

$$2^{3x+2} = 2^{-4}$$

$$\therefore 3x+2 = -4$$

$$3x = -6$$

$$x = -2$$

c) $4(6^{x+2}) = 144$

$$6^{x+2} = 36$$

$$6^{x+2} = 6^2$$

$$\therefore x+2 = 2$$

$$x = 0$$

d) $2^{x+2} - 2^{x+3} = -64$

$$2^{x+2}(1-2) = -64$$

$$2^{x+2}(-1) = -64$$

$$2^{x+2} = 64$$

$$2^{x+2} = 2^6$$

$$\therefore x+2 = 6$$

$$x = 4$$

e) $\frac{5^{x-1}}{25} = 125$

$$\frac{5^{x-1}}{5^2} = 5^3$$

$$5^{x-1} = 5^5$$

$$\therefore x-1 = 5$$

$$x = 6$$

Handout #3 p.269 16, 17a, b, d

p.270 5a

16. a) $P = \frac{1}{3} (1.1)^n$ # weeks
 ↑ previously assigned
 ↑ pond is $\frac{1}{3}$ covered initially
 ↑ 10% increase in coverage each wk

b) $M = M_0 \left(\frac{1}{2}\right)^{\frac{t}{4.5 \times 10^9}}$ number of years
 ↑ initial amt of U_{235}
 ↑ amt decreases by 50% in each half-life period

c) $I = 100(0.96)^n$

17. a) $P = 45000 (1.03)^n$ yrs since 1990

b) $P = 45000 (1.03)^{17}$
 ≈ 74378

∴ The city's popⁿ is going to be 74378 people in 2007

d) $90000 = 45000 (b)^{10}$

$2 = b^{10}$

$b = \sqrt[10]{2}$

$b \approx 1.072$

∴ If it took 10 yrs for the popⁿ to double the growth rate would need to be 7.2%.

p.270

5. a) $P = 2000000 (1.04)^n$
 or $P = 2 (1.04)^n$ where the popⁿ is in millions

Handout #4

a) $f(x) = 2(3)^{-x} - 18$

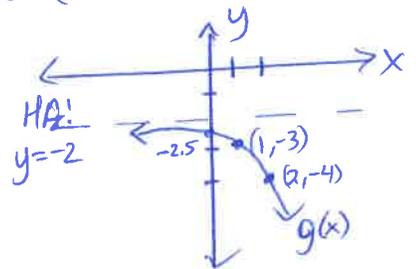
$x\text{-int, } f(x)=0$	$f(x)\text{-int, } x=0$
$18 = 2(3)^{-x}$	$f(0) = 2(3)^0 - 18$
$9 = 3^{-x}$	$= 2(1) - 18$
$3^2 = 3^{-x}$	$= -16$
$\therefore -x = 2$	$\therefore (0, -16)$
$x = -2$	
$\therefore (-2, 0)$	

b) $g(x) = -\left(\frac{1}{2}\right)^{-x+1} - 2$
 $= -\left(\frac{1}{2}\right)^{-(x-1)} - 2$

$(x, y) \rightarrow (-x+1, -y-2)$
 $(-1, 2) \rightarrow (2, -4)$
 $(0, 1) \rightarrow (1, -3)$
 $(1, \frac{1}{2}) \rightarrow (0, -2.5)$

Base function:

$y = \left(\frac{1}{2}\right)^x$



c) Text p.253 14

To transform $m(x) = -\left(\frac{3}{2}\right)^{2x-2}$ or $m(x) = -\left(\frac{3}{2}\right)^{2(x-1)}$ to $n(x) = -\left(\frac{9}{4}\right)^{-x+1} + 2$
 $= -\left(\frac{3}{2}\right)^{-2x+2} + 2$
 $= -\left(\frac{3}{2}\right)^{-2(x-1)} + 2$

you would need to reflect it over the y-axis and translate it up 2 units

Handout # 5

$$\begin{aligned} \text{a) } A &= P(1+i)^n \\ &= 6500(1.0085)^{24} \\ &\approx \$7964.07 \end{aligned}$$

∴ The BIC is worth
\$7964.07 after 6 yrs.

$$\begin{aligned} \text{b) } A &= P(1+i)^n \\ 3165.73 &= 3100\left(1 + \frac{r}{12}\right)^{12} \end{aligned}$$

$$\sqrt[12]{\frac{3165.73}{3100}} = 1 + \frac{r}{12}$$

$$r = 12\left(\sqrt[12]{\frac{3165.73}{3100}} - 1\right)$$

$$r \approx 0.021$$

∴ The annual interest rate is 2.1%.