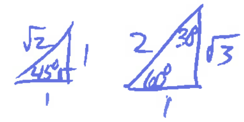
U5/L3 HW p.287 4–9, 11–15



4. a) $\sin 30^\circ \times \tan 60^\circ - \cos 30^\circ$

$= \frac{1}{2} \times \sqrt{3} - \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$

$= 0$

b) $2\cos 45^\circ \times \sin 45^\circ$

$= 2\left(\frac{1}{\sqrt{2}}\right) \times \frac{1}{\sqrt{2}}$

$= \cancel{2}^1\left(\frac{1}{\cancel{2}_1}\right)$

$= 1$

c) $\tan^2 30^\circ - \cos^2 45^\circ$

$= \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{3} - \frac{1}{2}$

$= \frac{2}{6} - \frac{3}{6}$

$= -\frac{1}{6}$

d) $1 - \frac{\sin 45^\circ}{\cos 45^\circ}$

$= 1 - \frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}}$

$= 1 - \frac{1}{\cancel{\sqrt{2}}_1} \cdot \frac{\cancel{\sqrt{2}}^1}{1}$

$= 1 - 1$

$= 0$

5. Prove $\sin^2\theta + \cos^2\theta = 1$

a) $\theta = 30^\circ$

$L.S. = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} + \frac{3}{4}$

$= 1$

$= R.S.$

b) $\theta = 45^\circ$

$L.S. = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

$= R.S.$

c) $\theta = 60^\circ$

$L.S. = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

$= R.S.$

6. Prove $\frac{\sin\theta}{\cos\theta} = \tan\theta$

a) $\theta = 30^\circ$

$L.S. = \frac{1}{2} \div \frac{\sqrt{3}}{2}$

$= \frac{1}{\cancel{2}_1} \cdot \frac{\cancel{2}^1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

$R.S. = \tan 30^\circ$

$= \frac{1}{\sqrt{3}}$

$L.S. = R.S.$

b) $\theta = 45^\circ$

$L.S. = \frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}}$

$= \frac{1}{\cancel{\sqrt{2}}_1} \cdot \frac{\cancel{\sqrt{2}}^1}{1}$

$= 1$

$R.S. = \tan 45^\circ$

$= 1$

$L.S. = R.S.$

c) $\theta = 60^\circ$

$L.S. = \frac{\sqrt{3}}{2} \div \frac{1}{2}$

$= \frac{\sqrt{3}}{\cancel{2}_1} \cdot \frac{\cancel{2}^1}{1}$

$= \sqrt{3}$

$R.S. = \tan 60^\circ$

$= \sqrt{3}$

$L.S. = R.S.$

7. $0^\circ \le \theta \le 90^\circ$

a) $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 60^\circ$

b) $\sqrt{3}\tan\theta = 1$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ$

c) $2\sqrt{2}\cos\theta = 2$

$\cos\theta = \frac{\cancel{2}^{1}}{{}^{1}\cancel{2}\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

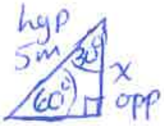
$\theta = 45^\circ$

d) $2\cos\theta = \sqrt{3}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30^\circ$

8.



Let x rep. the height of the top of the ladder above the floor.

$\sin 60^\circ = \frac{x}{5}$

$x = 5\sin 60^\circ$

$= \frac{5\sqrt{3}}{2}$

∴ The top of the ladder is exactly $\frac{5\sqrt{3}}{2}$ m above the floor. (assuming wall is ⊥ to the floor)

9.

~~Take up~~

$L.S. = \tan 30 + \frac{1}{\tan 30^\circ}$

$= \frac{1}{\sqrt{3}} + 1 \div \frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}} + 1 \cdot \frac{\sqrt{3}}{1}$

could just go to here i.e. $\cot 30^\circ = \sqrt{3}$

$= \frac{1}{\sqrt{3}} + \sqrt{3}$

common denom

$= \frac{1}{\sqrt{3}} + \frac{3}{\sqrt{3}}$

$= \frac{4}{\sqrt{3}}$

$R.S. = \frac{1}{\sin 30^\circ \cos 30^\circ}$

$= 1 \div \left(\frac{1}{2} \cdot \frac{\sqrt{3}}{2}\right)$

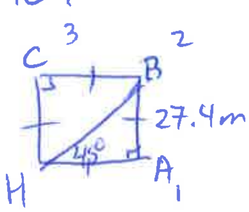
$= 1 \div \frac{\sqrt{3}}{4}$

$= 1 \cdot \frac{4}{\sqrt{3}}$

$= \frac{4}{\sqrt{3}}$

$L.S. = R.S.$

10.



a) The diagonal from Home plate to 2nd base creates 2 special triangles each with 45°, 45° and 90° angles. This means the sine ratio can be used to find the diagonal's length

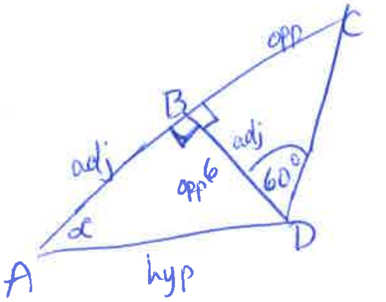
b) $\sin 45^\circ = \frac{27.4}{HB}$

$HB = 27.4 \div \frac{1}{\sqrt{2}}$

$HB \doteq 38.7$

∴ The dist. from home plate to 2nd base is approx. 38.7 m

11. a) (i) $\because \tan\alpha = 1$ and $\alpha = 45°$



$\therefore AB = BD$
$= 6$ units

(ii) $\tan 60° = \dfrac{BC}{6}$

$6\sqrt{3} = BC$

$BC = 6\sqrt{3}$

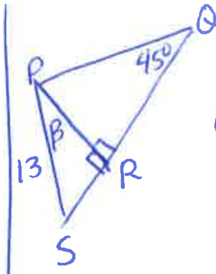
(iii) $A = \dfrac{bh}{2}$

$= \dfrac{(6\sqrt{3}+6)(\cancel{6})^{3}}{\cancel{2}_{1}}$

$= 3(6\sqrt{3}+6)$ units^2

11. b) (i) $\because \cos\beta = \dfrac{\sqrt{3}}{2}$

$\therefore \beta = 30°$



(ii) $\sin 30° = \dfrac{RS}{13}$ $\qquad$ $\cos 30° = \dfrac{PR}{13}$

$RS = 13\left(\dfrac{1}{2}\right)$ $\qquad$ $PR = 13\left(\dfrac{\sqrt{3}}{2}\right)$

$= \dfrac{13}{2}$ $\qquad$ $PR = \dfrac{13\sqrt{3}}{2}$

(iii) $\because \angle Q = 45°$

$\therefore PR = QR$

$QR = \dfrac{13\sqrt{3}}{2}$

iv) $A = \dfrac{bh}{2}$

$= \left(\dfrac{13}{2} + \dfrac{13\sqrt{3}}{2}\right)\left(\dfrac{13\sqrt{3}}{2}\right) \div 2$

$= \left(\dfrac{13+13\sqrt{3}}{2}\right)\left(\dfrac{13\sqrt{3}}{4}\right)$

$= \dfrac{169\sqrt{3} + 169(3)}{8}$

$= \dfrac{169}{8}(\sqrt{3}+3)$ square units

12. a) calculator
$\sin 45°(1-\cos 30°) + 5\tan 60°(\sin 60° - \tan 30°)$
$\doteq 2.595$

b) $\dfrac{\sqrt{2}}{2}\left(1 - \dfrac{\sqrt{3}}{2}\right) + 5\left(\dfrac{\sqrt{3}}{1}\right)\left(\dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{3}}{3}\right)$

$= \dfrac{\sqrt{2}}{2}\left(\dfrac{2-\sqrt{3}}{2}\right) + 5\sqrt{3}\left(\dfrac{3\sqrt{3}-2\sqrt{3}}{6}\right)$

$= \dfrac{2\sqrt{2}-\sqrt{6}}{4} + 5\sqrt{3}\left(\dfrac{\sqrt{3}}{6}\right)$

$= \dfrac{2\sqrt{2}-\sqrt{6}}{4} + \dfrac{5(\cancel{3})^{1}}{\cancel{6}_{2}}$

$= \dfrac{2\sqrt{2}-\sqrt{6}+10}{4}$

c) Megan won the prize for demonstrating more math skills with her exact sol^n in (b) than Louise did using her calc. in (a)

13. If $\cot\alpha = \sqrt{3}$, then $\alpha = 30°$

$(\sin\alpha)(\cot\alpha) - \cos^2\alpha$
$= (\sin 30°)(\cot 30°) - \cos^2 30°$
$= \dfrac{1}{2}(\sqrt{3}) - \left(\dfrac{\sqrt{3}}{2}\right)^2$
$= \dfrac{\sqrt{3}}{2} - \dfrac{3}{4}$
$= \dfrac{2\sqrt{3}-3}{4}$

14. If $\csc\beta = 2$, then $\beta = 30°$

$\dfrac{\tan 30°}{\sec 30°} - \sin^2 30°$

$= \dfrac{1}{\sqrt{3}} \div \dfrac{2}{\sqrt{3}} - \left(\dfrac{1}{2}\right)^2$

$= \dfrac{1}{\cancel{\sqrt{3}}_{1}} \times \dfrac{\cancel{\sqrt{3}}^{1}}{2} - \dfrac{1}{4}$

$= \dfrac{1}{2} - \dfrac{1}{4}$

$= \dfrac{2}{4} - \dfrac{1}{4}$

$= \dfrac{1}{4}$

15. Prove $1+\cot^2\theta = \csc^2\theta$

a) $\underline{\theta = 30^\circ}$

L.S. $= 1+(\sqrt{3})^2$ R.S. $= (2)^2$

$= 4$

$= 1+3$

$= 4$

L.S. = R.S.

b) $\underline{\theta = 45^\circ}$

L.S. $= 1+(1)^2$ R.S. $= (\sqrt{2})^2$

$= 1+1$ $= 2$

$= 2$

L.S. = R.S.

c) $\underline{\theta = 60^\circ}$

L.S. $= 1+\left(\frac{1}{\sqrt{3}}\right)^2$ R.S. $= \left(\frac{2}{\sqrt{3}}\right)^2$

$= 1+\frac{1}{3}$ $= \frac{4}{3}$

$= \frac{4}{3}$

L.S. = R.S.