U5/L7 Proving Identities (together)

a) $\frac{\cos x}{1-\sin x} + \frac{\cos x}{1+\sin x} = \frac{2}{\cos x}$

L.S. $= \frac{\cos x}{1-\sin x} + \frac{\cos x}{1+\sin x}$ comm. denom. R.S. $= \frac{2}{\cos x}$

$= \frac{\cos x(1+\sin x) + \cos x(1-\sin x)}{(1-\sin x)(1+\sin x)}$

$= \frac{\cos x + \cos x \cancel{\sin x} + \cos x - \cos x \cancel{\sin x}}{1-\sin^2 x}$ ← Pyth. Ident

$= \frac{2\cancel{\cos x}}{\cos^{\cancel{2}} x}$

$= \frac{2}{\cos x}$ L.S. = R.S.

□, QED

Steps

1. Write each side separately and usually we manipulate the most complicated side until it matches the expression on the other side.
2. Strategies that can be used include algebraic techniques, like comm. denom, factoring, expanding, expressing $\tan\theta$ or $\cot\theta$ expressions in terms of $\sin\theta$ and $\cos\theta$, and replacing expressions using the Pyth. Identity or its variations.

b) L.S. $= \frac{\sin x + \sin^2 x}{(\cos x)(1+\sin x)}$ CF R.S. $= \tan x$

$= \frac{\sin x\,(1+\cancel{\sin x})}{\cos x\,(1+\cancel{\sin x})}$

$= \tan x$

QED

c) L.S. $= \cot x$

R.S. $= \cos x \sin x + \cos^3 x \csc x$ CF of $\cos x$

$= \cos x(\sin x + \cos^2 x \csc x)$ change $\csc x$ to $\frac{1}{\sin x}$

$= \cos x\left(\sin x + \frac{\cos^2 x}{\sin x}\right)$ common denom

$= \cos x\left(\frac{\sin^2 x + \cos^2 x}{\sin x}\right)$ Pyth. Id

$= \cos x\left(\frac{1}{\sin x}\right)$

$= \frac{\cos x}{\sin x}$

$= \cot x$

□

U5/L7 HW

Handout Q's

a) $L.S. = \frac{4}{\cos^2 x} - 5$ $\quad R.S. = 4\tan^2 x - 1$ Quotient Id.

$= \frac{4 - 5\cos^2 x}{\cos^2 x}$

$= \frac{4\sin^2 x}{\cos^2 x} - 1$ Common denom

$= \frac{4\sin^2 x - \cos^2 x}{\cos^2 x}$ Pyth. Id.

$= \frac{4(1-\cos^2 x) - \cos^2 x}{\cos^2 x}$

$= \frac{4 - 4\cos^2 x - \cos^2 x}{\cos^2 x}$

$= \frac{4 - 5\cos^2 x}{\cos^2 x}$

QED

b) $L.S. = (\sin x - \cos x)(\sin x + \cos x)$ expand $\quad R.S. = 2\sin^2 x - 1$

$= \sin^2 x - \cos^2 x$ Pyth Id

$= \sin^2 x - (1 - \sin^2 x)$

$= \sin^2 x - 1 + \sin^2 x$

$= 2\sin^2 x - 1$

$\square$

Text p.310 5,8

#5.a) $L.S. = \frac{\sin x}{\tan x}$ $\quad R.S. = \cos x$

$= \sin x \div \frac{\sin x}{\cos x}$

$= \cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}}$

$= \cos x$

QED

b) $L.S. = \frac{\tan\theta}{\cos\theta}$ $\quad R.S. = \frac{\sin\theta}{1-\sin^2\theta}$

$= \frac{\sin\theta}{\cos\theta} \div \frac{\cos\theta}{1}$

$= \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$= \frac{\sin\theta}{\cos^2\theta}$

$= \frac{\sin\theta}{1-\sin^2\theta}$

$\square$

U5/L7

p. 310

5. c) L.S. $= \frac{1}{\cos\alpha} + \tan\alpha$ R.S. $= \frac{1+\sin\alpha}{\cos\alpha}$

$= \frac{1}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha}$

$= \frac{1+\sin\alpha}{\cos\alpha}$

L.S. = R.S.

d) L.S. $= 1-\cos^2\theta$ R.S. $= \sin\theta\cos\theta\tan\theta$

$= \sin\theta \cancel{\cos}\theta \frac{\sin\theta}{\cancel{\cos}\theta}$

$= \sin^2\theta$

$= 1-\cos\theta$

QED

8. a) L.S. $= \frac{\sin^2\phi}{1-\cos\phi}$ R.S. $= 1+\cos\phi$

$= \frac{1-\cos^2\phi}{1-\cos\phi}$

$= \frac{\cancel{(1-\cos\phi)}(1+\cos\phi)}{\cancel{1-\cos\phi}}$

$= 1+\cos\phi$

$\square$

b) L.S. $= \frac{\tan^2\alpha}{1+\tan^2\alpha}$ R.S. $= \sin^2\alpha$

$= \frac{\sin^2\alpha}{\cos^2\alpha} \div \left(1+\frac{\sin^2\alpha}{\cos^2\alpha}\right)$

$= \frac{\sin^2\alpha}{\cos^2\alpha} \div \left(\frac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha}\right)$

$= \frac{\sin^2\alpha}{\cancel{\cos^2\alpha}} \cdot \frac{\cancel{\cos^2\alpha}}{1}$

$= \sin^2\alpha$

L.S. = R.S.

8\. c) $L.S. = \cos^2 x$ $\qquad$ $R.S. = (1 - \sin x)(1 + \sin x)$

$$= 1 - \sin^2 x$$

$$= \cos^2 x$$

$\square$

d) $L.S. = \sin^2\theta + 2\cos^2\theta - 1$ $\qquad$ $R.S. = \cos^2\theta$

$$= (\cancel{1} - \cos^2\theta) + 2\cos^2\theta \cancel{- 1}$$

$$= \cos^2\theta$$

QED

e) $L.S. = \sin^4\alpha - \cos^4\alpha$ $\qquad$ $R.S. = \sin^2\alpha - \cos^2\alpha$

$$= (\sin^2\alpha + \cos^2\alpha)(\sin^2\alpha - \cos^2\alpha)$$

$$= (1)(\sin^2\alpha - \cos^2\alpha)$$

$L.S. = R.S.$

f) $L.S. = \tan\theta + \dfrac{1}{\tan\theta}$ $\qquad$ $R.S. = \dfrac{1}{\sin\theta\cos\theta}$

$$= \frac{\sin\theta}{\cos\theta} + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$\square$