

Unit 5 Review

Handout #1

Text p. 338 #2

2. a) $(\sin 45^\circ)(\cos 45^\circ) + (\sin 30^\circ)(\cos 60^\circ)$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{2}{4} + \frac{1}{4}$$

$$= \frac{3}{4}$$

b) $(1 - \tan 45^\circ)(\sin 30^\circ)(\cos 30^\circ)(\tan 60^\circ)$

$$= (1-1)\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{1}\right)$$

$$= 0 \left(\frac{3}{4}\right)$$

$$= 0$$

c) $\tan 30^\circ + 2(\sin 45^\circ)(\cos 60^\circ)$

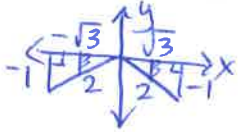
$$= \frac{\sqrt{3}}{3} + 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{3} + \frac{\sqrt{2}}{2}$$

$$= \frac{2\sqrt{3} + 3\sqrt{2}}{6}$$

p340 #2

a) $\sin \theta = -\frac{1}{2}$



$$\beta = 30^\circ$$

In Quad III
 $\theta = 180^\circ + 30^\circ$
 $\theta = 210^\circ$

In Quad IV
 $\theta = 360^\circ - 30^\circ$
 $\theta = 330^\circ$

b) $\cos \theta = \frac{\sqrt{3}}{2}$

In Quad I
 $\theta = 30^\circ$

In Quad IV
 $\theta = 360^\circ - 30^\circ$
 $\theta = 330^\circ$

c) $\cot \theta = -1$
 $\tan \theta = -1$

In Quad II
 $\theta = 180^\circ - 45^\circ$
 $\theta = 135^\circ$

In Quad IV
 $\theta = 360^\circ - 45^\circ$
 $\theta = 315^\circ$

d) $\sec \theta = -\frac{2}{1}$
 $\cos \theta = -\frac{1}{2}$

$$\beta = 60^\circ$$

In Quad II
 $\theta = 180^\circ - 60^\circ$
 $\theta = 120^\circ$

In Quad III
 $\theta = 180^\circ + 60^\circ$
 $\theta = 240^\circ$

Handout #2

a) $\sin 30^\circ = \frac{1}{2}$

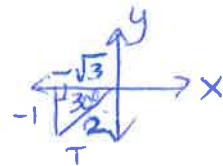
b) $\cos 45^\circ = \frac{\sqrt{2}}{2}$

c) $\csc 60^\circ = \frac{1}{\sin 60^\circ}$

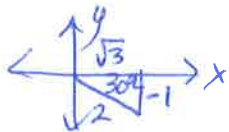
$$= \frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}$$

d) $\tan 210^\circ$

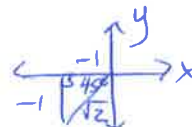
$$= -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$



e) $\sin 330^\circ = -\frac{1}{2}$

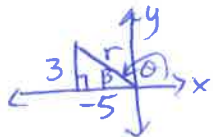


f) $\cos 225^\circ = -\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}$



Handout #3

a) $(-5, 3)$



$$r = \sqrt{34}$$

$$\sin \theta = \frac{3\sqrt{34}}{34}$$

$$\cos \theta = \frac{-5\sqrt{34}}{34}$$

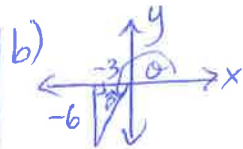
$$\tan \theta = -\frac{3}{5}$$

$$\beta = \tan^{-1}\left(\frac{3}{5}\right)$$

$$\beta = 31^\circ$$

$$\therefore \theta = 180^\circ - 31^\circ$$

$$\theta = 149^\circ$$



$$r = \sqrt{45}$$

$$r = 3\sqrt{5}$$

$$\sin \theta = \frac{-3}{3\sqrt{5}} \text{ or } -\frac{2\sqrt{5}}{5}$$

$$\cos \theta = \frac{-6}{3\sqrt{5}} \text{ or } -\frac{\sqrt{5}}{5}$$

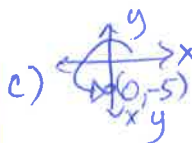
$$\tan \theta = \frac{-6}{-3} \text{ or } 2$$

$$\beta = \tan^{-1} 2$$

$$\beta = 63^\circ$$

$$\therefore \theta = 180^\circ + 63^\circ$$

$$\theta = 243^\circ$$



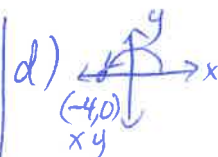
$$r = 5$$

$$\sin \theta = \frac{-5}{5} \text{ or } -1$$

$$\cos \theta = \frac{0}{5} \text{ or } 0$$

$$\tan \theta = \frac{-5}{0} \text{ or } \phi$$

$$\theta = 270^\circ$$



$$r = 4$$

$$\sin \theta = \frac{0}{4} \text{ or } 0$$

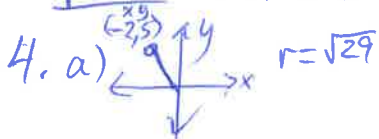
$$\cos \theta = \frac{-4}{4} \text{ or } -1$$

$$\tan \theta = \frac{0}{-4} \text{ or } 0$$

$$\theta = 180^\circ$$

Handout #4

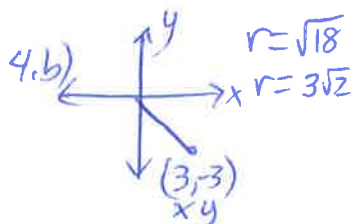
p. 338 4, 5, 7, 10, 11



$$\sin \theta = \frac{5\sqrt{29}}{29}$$

$$\cos \theta = \frac{-2\sqrt{29}}{29}$$

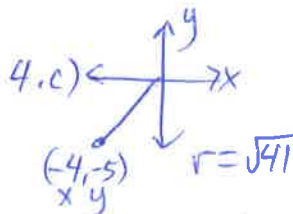
$$\tan \theta = -\frac{5}{2}$$



$$\sin \theta = \frac{-3}{3\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{3}{3\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$

$$\tan \theta = \frac{-3}{3} \text{ or } -1$$



$$\sin \theta = \frac{-5\sqrt{41}}{41}$$

$$\cos \theta = \frac{-4\sqrt{41}}{41}$$

$$\tan \theta = \frac{-5}{-4} \text{ or } \frac{5}{4}$$

5. $\cos \phi = \frac{-7}{\sqrt{53}}$, $y^2 = (\sqrt{53})^2 - (-7)^2 = 53 - 49 = 4$

a) The terminal arm lies in quad II or III $y = \pm 2$

b) Quad II

$$\sin \phi = \frac{2}{\sqrt{53}} \quad \csc \phi = \frac{\sqrt{53}}{2}$$

$$\tan \phi = -\frac{2}{7} \quad \cot \phi = -\frac{7}{2}$$

$$\sec \phi = -\frac{\sqrt{53}}{7}, \quad \theta = 180^\circ - 16^\circ$$

Quad III

$$\sin \phi = \frac{-2}{\sqrt{53}} \quad \csc \phi = \frac{-\sqrt{53}}{2}$$

$$\tan \phi = \frac{2}{7} \quad \cot \phi = \frac{7}{2}$$

$$\sec \phi = -\frac{\sqrt{53}}{7}, \quad \theta = 180^\circ + 16^\circ$$

$$\beta = \tan^{-1}\left(\frac{2}{7}\right)$$

$$\beta = 16^\circ$$

7. a) L.S. = $\tan \alpha \cos \alpha$
 $= \frac{\sin \alpha}{\cos \alpha} \cdot \cos \alpha$
 $= \sin \alpha$

R.S. = $\sin \alpha$ b) L.S. = $\frac{1}{\cot \phi}$
 $= \tan \phi$

R.S. = $\sin \phi \sec \phi$
 $= \sin \phi \cdot \frac{1}{\cos \phi}$
 $= \frac{\sin \phi}{\cos \phi}$
 $= \tan \phi$

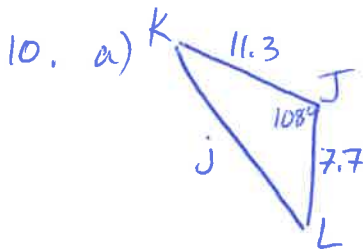
□

QED

c) L.S. = $1 - \cos^2 x$ R.S. = $\frac{\sin x \cos x}{\cot x}$
 $= \sin x \cos x \cdot \frac{\cos x}{\sin x}$
 $= \sin x \cos x \cdot \frac{\sin x}{\cos x}$
 $= \sin^2 x$
 $= 1 - \cos^2 x$
 L.S. = R.S.

d) L.S. = $\sec \theta \cos \theta + \sec \theta \sin \theta$ R.S. = $1 + \tan \theta$
 $= \frac{1}{\cos \theta} \cdot \cos \theta + \frac{1}{\cos \theta} \cdot \sin \theta$
 $= 1 + \frac{\sin \theta}{\cos \theta}$
 $= 1 + \tan \theta$

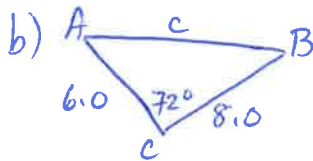
□



$$j^2 = l^2 + k^2 - 2lk \cos J$$

$$j^2 = 11.3^2 + 7.7^2 - 2(11.3)(7.7) \cos 108^\circ$$

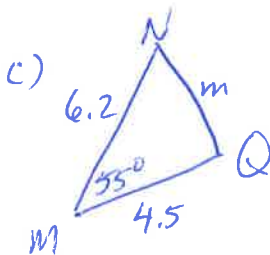
$j \approx 15.5$ units



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 8^2 + 6^2 - 2(8)(6) \cos 72^\circ$$

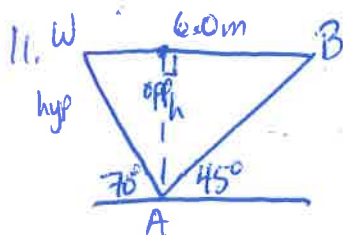
$c \approx 8.4$ units



$$m^2 = n^2 + q^2 - 2nq \cos 55^\circ$$

$$m^2 = 4.5^2 + 6.2^2 - 2(4.5)(6.2) \cos 55^\circ$$

$m \approx 5.2$ units



Let h rep the height from the floor to the ceiling.

$\angle W = 70^\circ$ (AA)
 (i) $\angle WAB = 65^\circ$ (supp \angle s)
 $\angle B = 45^\circ$ (AA)

(iii) $\sin 70^\circ = \frac{h}{4.68}$

(ii) $\frac{AW}{\sin B} = \frac{BW}{\sin WAB}$

$h \approx 4.4$

$\frac{AW}{\sin 45^\circ} = \frac{6.0}{\sin 65^\circ}$

$AW \approx 4.68$ m

∴ The ceiling is approx 4.4 m high.

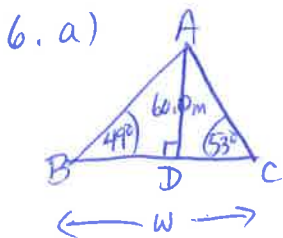
Handout #5

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3. $\cos \theta = \frac{-5}{13}$, quad II

$$x = -5, r = 13, y = \sqrt{169 - 25} = \sqrt{144} = 12$$

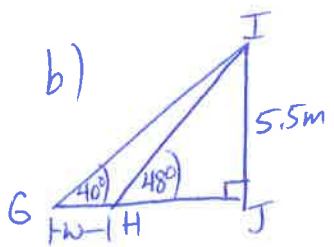
a) $\sin \theta \cos \theta = \frac{12}{13} \left(\frac{-5}{13} \right) = \frac{-60}{169}$
 b) $\cot \theta \tan \theta = \frac{-5}{12} \left(\frac{12}{-5} \right) = 1$



(i) $\tan 53^\circ = \frac{60.0}{CD}$
 $CD \doteq 45.21 \text{ m}$

(iii) $w = BD + CD = 52.16 + 45.21 = 97.4 \text{ m}$

(ii) $\tan 49^\circ = \frac{60.0}{BD}$
 $BD \doteq 52.16 \text{ m}$

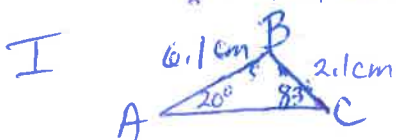


(i) $\tan 48^\circ = \frac{5.5}{HJ}$
 $HJ \doteq 4.95 \text{ m}$

(ii) $\tan 40^\circ = \frac{5.5}{GJ}$
 $GJ \doteq 6.55 \text{ m}$
 (iii) $w = GJ - HJ = 6.55 - 4.95 = 1.6 \text{ m}$

7. a) $a = 1.5 \text{ cm}, b = 2.8 \text{ cm}, \angle A = 41^\circ$
 $\therefore b \sin A = 1.8$ and $a < b \sin A$
 \therefore no triangle

b) $a = 2.1 \text{ cm}, c = 6.1 \text{ cm}, \angle A = 20^\circ$
 $\therefore c \sin A = 2.08$ and $c \sin A < a < c \therefore$ 2 triangles



(i) $\frac{\sin C}{6.1} = \frac{\sin 20^\circ}{2.1}$
 $\angle C \doteq 83^\circ$

(i) $\angle C = 180^\circ - 83^\circ = 97^\circ$
 $\angle C \doteq 97^\circ$

(ii) $\angle B = 77^\circ$ (AST)

(ii) $\angle B \doteq 63^\circ$ (AST)

(iii) $\frac{b}{\sin 77^\circ} = \frac{2.1}{\sin 20^\circ}$
 $b \doteq 6.0 \text{ cm}$

(iii) $\frac{b}{\sin 63^\circ} = \frac{2.1}{\sin 20^\circ}$
 $b \doteq 5.5 \text{ cm}$

Handout #6

$$\begin{aligned}
 \text{a) L.S.} &= \frac{\cos x - \sin x - \cos^3 x}{\cos x} \\
 &= 1 - \tan x - \cos^2 x \\
 &= \sin^2 x - \tan x \quad \square
 \end{aligned}$$

$$\text{R.S.} = \sin^2 x - \tan x$$

$$\begin{aligned}
 \text{b) L.S.} &= \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} \\
 &= \frac{1 + \sin x - (1 - \sin x)}{(1 - \sin x)(1 + \sin x)} \\
 &= \frac{1 + \sin x - 1 + \sin x}{1 - \sin^2 x} \\
 &= \frac{2 \sin x}{\cos^2 x} \\
 &= \frac{2 \sin x}{\cos x} \cdot \frac{1}{\cos x} \\
 &= 2 \tan x \cdot \frac{1}{\cos x}
 \end{aligned}$$

$$\text{R.S.} = \frac{2 \tan x}{\cos x}$$

$$= \frac{2 \tan x}{\cos x} \quad \text{Q.E.D.}$$

$$\begin{aligned}
 \text{c) L.S.} &= \cos^2 x - \cos^4 x \\
 &= \cos^2 x (1 - \cos^2 x) \\
 &= \cos^2 x \cdot \sin^2 x
 \end{aligned}$$

$$\text{R.S.} = \cos^2 x \sin^2 x$$

$$\text{L.S.} = \text{R.S.}$$

Handout #7 Quad III \square

$$\text{a) } \sin x = -\frac{1}{2} \leftarrow y$$

$$y = -1, r = 2, x = -\sqrt{3}$$

$$\tan^2 x - \cos x + \csc x$$

$$= \left(\frac{-1}{-\sqrt{3}}\right)^2 - \left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{-2}{1}\right)$$

$$= \frac{1}{3} + \frac{\sqrt{3}}{2} - \frac{2}{1}$$

$$= \frac{2 + 3\sqrt{3} - 12}{6}$$

$$= \frac{3\sqrt{3} - 10}{6}$$

$$\text{b) } \cot x = \frac{3}{1} \leftarrow x \begin{matrix} (-3) \\ (-1) \end{matrix}$$

$$x = -3, y = -1, r = \sqrt{10}$$

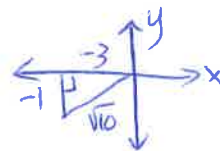
$$\tan^2 x - \cos x + \csc x$$

$$= \left(\frac{-1}{-3}\right)^2 + \frac{3}{\sqrt{10}} - \frac{\sqrt{10}}{1}$$

$$= \frac{1}{9} + \frac{3\sqrt{10}}{10} - \frac{\sqrt{10}}{1}$$

$$= \frac{10 + 27\sqrt{10} - 90\sqrt{10}}{90}$$

$$= \frac{10 - 63\sqrt{10}}{90}$$



$$\begin{aligned}
 r^2 &= x^2 + y^2 \\
 r &= \sqrt{(-3)^2 + (-1)^2} \\
 r &= \sqrt{10}
 \end{aligned}$$

