U7/L1 p.424 4,6 p.447 1, 2a,b

p.424

#4. 85, 102, 119, ...

$a=85, d=17$

$t_n = a+(n-1)d$

$t_n = 85+17(n-1)$

$t_n = 85+17n-17$

$\boxed{t_n = 17n+68}$

$t_{15} = 17(15)+68$

$\boxed{t_{15} = 323}$

#6. a) $t_1=19, d=8$

$\therefore$ recursive formula is $t_1=19$, $t_n=t_{n-1}+8, n>1$

and the gen. term is $t_n=19+8(n-1)$

or $t_n=8n+11$

6.b) $t_1=4, d=-5$

$\therefore t_1=4, t_n=t_{n-1}-5, n>1$

and $t_n = 4-5(n-1)$

or $t_n=-5n+9$

6.c) $t_1=21$, $t_2=26$

$\therefore d=5$

and $t_1=21$, $t_n=t_{n-1}+5, n>1$

and $t_n = 21+5(n-1)$

or $t_n = 5n+16$

6.d) $t_4=35$, $d=-12$

$\therefore t_1 = 35+3(12)$

$\boxed{t_1=71}$

and recurs. form. is $t_1=71$, $t_n=t_{n-1}-12, n>1$

and gen. term is $t_n=71-12(n-1)$

or $t_n=-12n+83$

p.447

#1. a) 29, 21, 13, ... $a=29, d=-8$

(i) $t_1=29$, $t_n=t_{n-1}-8, n>1$

(ii) $t_n=a+(n-1)d$

$t_n=29-8(n-1)$

$\boxed{t_n=-8n+37}$

(iii) $t_{10}=-8(10)+37$

$\boxed{t_{10}=-43}$

1.b) −8, −16, −24, ... $a=-8, d=-8$

(i) $t_1=-8$, $t_n=t_{n-1}-8, n>1$

(ii) $t_n=-8-8(n-1)$

$\boxed{t_n=-8n}$

(iii) $t_{10}=-8(10)$

$\boxed{t_{10}=-80}$

p.447

1.c) $-17, -9, -1, \ldots$ $a=-17, d=8$

(i) $t_1 = -17,\ t_n = t_{n-1} + 8,\ n>1$

(ii) $t_n = -17 + 8(n-1)$

$\boxed{t_n = 8n - 25}$

(iii) $t_{10} = 8(10) - 25$

$\boxed{t_{10} = 55}$

1.d) $3.25, 9.5, 15.75, \ldots$ $a=3.25, d=6.25$

(i) $t_1 = 3.25,\ t_n = t_{n-1} + 6.25,\ n>1$

(ii) $t_n = 3.25 + 6.25(n-1)$

$\boxed{t_n = 6.25n - 3}$

(iii) $t_{10} = 6.25(10) - 3$

$\boxed{t_{10} = 59.5}$

1.e) $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}, \ldots$

Think: $\frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \ldots$

$a = \frac{1}{2},\ d = \frac{1}{6}$

(i) $t_1 = \frac{1}{2},\ t_n = t_{n-1} + \frac{1}{6},\ n>1$

(ii) $t_n = \frac{1}{2} + \frac{1}{6}(n-1)$

$t_n = \frac{3}{6} + \frac{1}{6}n - \frac{1}{6}$

$\boxed{t_n = \frac{1}{6}n + \frac{1}{3}}$

(iii) $t_{10} = \frac{1}{\cancel{6}_3}(\cancel{10})^5 + \frac{1}{3}$

$\boxed{t_{10} = \frac{6}{3} \text{ or } 2}$

1.f) $x,\ 3x+3y,\ 5x+6y, \ldots$ $a = x$, $d = 2x+3y$

(i) $t_1 = x,\ t_n = t_{n-1} + 2x + 3y,\ n>1$

(ii) $t_n = x + (2x+3y)(n-1)$

$t_n = x + 2xn - 2x + 3ny - 3y$

$t_n = -x + 2xn + 3ny - 3y$

$\boxed{t_n = x(2n-1) + 3y(n-1)}$

(iii) $t_{10} = x[2(10)-1] + 3y(10-1)$

$\boxed{t_{10} = 19x + 27y}$

2. a) $t_1 = 17,\ d = 11$

$\therefore t_1 = 17,\ t_n = t_{n-1} + 11,\ n>1$

and $t_n = 17 + 11(n-1)$

$\boxed{t_n = 11n + 6}$

2.b) $t_1 = 38,\ d = -7$

$\therefore t_1 = 38,\ t_n = t_{n-1} - 7,\ n>1$

and $t_n = 38 - 7(n-1)$

$\boxed{t_n = -7n + 45}$

U7/L1/P2 p.430 3, 4, 7, 8, 9 a, b, c 10 a, b, d 11, 12

p.447 4 a, c, e, f 5 c, d, e

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4\. a) 15, 30, 45, ...

(i) ∵ $d = -15$ ∴ arithmetic

(ii) R.F. $t_1 = 15$, $t_n = t_{n-1} + 15$, $n > 1$

G.T. $t_n = a + (n-1)d$

$t_n = 15 + 15(n-1)$

$\boxed{t_n = 15n}$

$t_6 = 15(6)$

$\boxed{t_6 = 90}$

4\. c) 23, −46, 92, ...

(i) ∵ $r = -2$ ∴ geometric

(ii) R.F. $t_1 = 23$, $t_n = -2t_{n-1}$, $n > 1$

G.T. $\boxed{t_n = 23(-2)^{n-1}}$

$t_6 = 23(-2)^{6-1}$

$t_6 = 23(-2)^5$

$\boxed{t_6 = -736}$

4\. e) 3.8, 5, 6.2, ...

(i) ∵ $d = 1.2$ ∴ arithmetic

(ii) R.F $t_1 = 3.8$, $t_n = t_{n-1} + 1.2$, $n > 1$

G.T. $t_n = 3.8 + 1.2(n-1)$

$\boxed{t_n = 1.2n + 2.6}$

$t_6 = 1.2(6) + 2.6$

$\boxed{t_6 = 9.8}$

4\. f) $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}$

(i) ∵ $r = \frac{2}{3}$ ∴ geometric

(ii) R.F. $t_1 = \frac{1}{2}$, $t_n = \frac{2}{3}t_{n-1}$, $n > 1$

$\boxed{t_n = \frac{1}{2}\left(\frac{2}{3}\right)^{n-1}}$

$t_6 = \frac{1}{2}\left(\frac{2}{3}\right)^5$

$t_6 = \frac{1}{\cancel{2}_1}\left(\frac{\cancel{32}^{16}}{243}\right)$

$\boxed{t_6 = \frac{16}{243}}$

Think: $\frac{9}{18}, \frac{6}{18}, \frac{4}{18}, \ldots$

and $\frac{\cancel{6}^2}{\cancel{18}_1} \cdot \frac{\cancel{18}^1}{\cancel{9}_3}$ or $\frac{\cancel{4}^2}{\cancel{18}_1} \cdot \frac{\cancel{18}^1}{\cancel{6}_3}$

$= \frac{2}{3}$ $= \frac{2}{3}$

5\. c) $t_1 = 5$, $t_n = t_{n-1} - 12$, $n > 1$

(i) arithmetic

(ii) First 5 terms: 5, −7, −19, −31, −43

5\. d) $t_1 = -2$, $\frac{t_n}{t_{n-1}} = -2$, $n > 1$

(i) geometric

$r = -2$ so mult terms by −2

(ii) First 5 terms: −2, 4, −8, 16, −32

5\. e) $t_1 = 8$, $t_2 = 11$, $t_n = 2t_{n-1} - t_{n-2}$, $n > 2$

(i) arithmetic (involves subtraction, not division - which is for geometric)

(ii) First 5 terms:

8, 11, 14, 17, 20

t_n can also be expressed as $t_n = t_{n-1} + 3$

<u>Can check:</u>

$t_3 = 2t_2 - t_1$

$= 2(11) - 8$

$= 22 - 8$

$= 14$

U7/L1 Arith. & Geom. Sequences

p.430 7, 8 then 3, 4, 9a, b, c 10a, b, d 11, 12

p430

7. a) (i) 9, 13, 17, 21, ...
$\because d = 4 \therefore$ arithmetic

(ii) $t_n = a + (n-1)d$

$t_n = 9 + 4(n-1)$

$\boxed{t_n = 4n + 5}$

b) (i) 7, −21, 63, −189, ...
$\because r = -3 \therefore$ geometric

(ii) $t_n = ar^{n-1}$

$\boxed{t_n = 7(-3)^{n-1}}$

c) (i) 18, −18, 18, −18, ...
$\because r = -1 \therefore$ geometric

(ii) $t_n = ar^{n-1}$

$\boxed{t_n = 18(-1)^{n-1}}$

d) (i) 31, 32, 34, 37, ...
neither

e) (i) 29, 19, 9, −1, ...
$\because d = -10 \therefore$ arithmetic

(ii) $t_n = a + (n-1)d$

$t_n = 29 - 10(n-1)$

$\boxed{t_n = -10n + 39}$

f) (i) 128, 96, 72, 54, ...
$\because r = \frac{3}{4} \therefore$ geometric

(ii) $t_n = ar^{n-1}$

$\boxed{t_n = 128\left(\frac{3}{4}\right)^{n-1}}$

Geometric → 8. a) the first term is 19 and the common ratio is 5

$\therefore t_1 = 19,\ t_n = 5t_{n-1},\ n > 1$ is the recursive formula

and the general term is $t_n = 19(5)^{n-1}$

b) $t_1 = -9$ and $r = -4$

$\therefore$ The recursive formula is $t_n = -4t_{n-1},\ t_1 = -9,\ n > 1$

and the general term is $t_n = -9(-4)^{n-1}$

c) first term is 144 and the second term is 36

(i) $r = \frac{36}{144}$

$= \frac{1}{4}$

(ii) $\therefore$ The recursive formula is $t_1 = 144,\ t_n = \frac{1}{4}t_{n-1},\ n > 1$

and the general term is $t_n = 144\left(\frac{1}{4}\right)^{n-1}$

d) $t_1 = 900$ and $r = \frac{1}{6}$

$\therefore$ The recursive formula is $t_1 = 900,\ t_n = \left(\frac{1}{6}\right)t_{n-1},\ n > 1$ and

the general term is $t_n = 900\left(\frac{1}{6}\right)^{n-1}$

p. 430 3, 4 9 a, b, c 10 a, b, d 11, 12

\# 3. $t_{31} = 123$, $t_{32} = 1107$, geometric

$r = \frac{1107}{123}$

$r = 9$

$\therefore t_n = 9t_{n-1}$ so $t_{33} = 9t_{32}$

$= 9(1107)$

$= 9963$

\#4. 1 813 985 280, 302 330 880, 50 388 480, ... geometric

$r = \frac{302\,330\,880}{1\,813\,985\,280}$

$= \frac{1}{6}$ (or $0.1\dot{6}$)

$t_n = ar^{n-1}$

$t_{10} = 1\,813\,985\,280\left(\frac{1}{6}\right)^9$

$t_{10} = 180$

9. a) $t_1 = 18$, $t_n = \left(\frac{2}{3}\right)^{n-1} \cdot t_{n-1}$, $n > 1$

(i) $t_2 = \left(\frac{2}{\cancel{3}_1}\right)^1 \cdot \cancel{18}^{6}$

$\boxed{t_2 = 12}$

$t_3 = \left(\frac{2}{3}\right)^2 \cdot 12$

$t_3 = \frac{4}{\cancel{9}_3} \cdot \cancel{12}^4$

$\boxed{t_3 = \frac{16}{3}}$

(ii) $\frac{t_2}{t_1} = \frac{12}{18}$

$\boxed{\frac{t_2}{t_1} = \frac{2}{3} \text{ or } \frac{6}{9}}$

$\frac{t_3}{t_2} = \frac{16}{3} \div 12$

$\frac{t_3}{t_2} = \frac{\cancel{16}^4}{3} \times \frac{1}{\cancel{12}_3}$

$\boxed{\frac{t_3}{t_2} = \frac{4}{9}}$

$\because$ The quotients are not the same

$\therefore$ there isn't a common ratio, so the sequence is not geometric.

9. b) $t_1 = -8$, $t_n = -3t_{n-1}$, $n > 1$

(i) geometric with $r = -3$

(ii) First five terms: $-8, 24, -72, 216, -648$

9. c) $t_1 = 123$, $t_n = \frac{t_{n-1}}{3}$, $n > 1$

(i) geometric with $r = \frac{1}{3}$

(ii) First 5 terms: $123, 41, \frac{41}{3}, \frac{41}{9}, \frac{41}{27}$

p. 430 10 a, b, d 11, 12

10. a) $t_n = 4^n$

$t_1 = 4, t_2 = 16, t_3 = 64, t_4 = 256, t_5 = 1024$ and $\frac{t_n}{t_{n-1}} = 4$

$\because r = 4$

$\therefore$ geometric

b) $t_n = 3^n + 5$

$t_1 = 8, t_2 = 14, t_3 = 32$

$\frac{t_2}{t_1} = 1.75$ or $\frac{7}{4}$ | $\frac{t_3}{t_2} = \frac{32}{14}$ or $\frac{16}{7}$

$\because$ no common ratio

$\therefore$ not geometric

d) $t_n = 7 \times (-5)^{n-4}$

$t_1 = 7(-5)^{-3}$
$= \frac{-7}{125}$

$t_2 = 7(-5)^{2}$
$= \frac{7}{25}$

$t_3 = 7(-5)^{-1}$
$= -\frac{7}{5}$

$t_4 = 7(-5)^0$
$= 7$

$t_5 = 7(-5)$
$= -35$

$\because r = -5$

$\therefore$ geometric

11. $t_5 = 45, t_8 = 360$, find 20^{th} term of geom. sequence.

(i) ① $45 = ar^4$ ② $360 = ar^7$

② ÷ ①

$\frac{360}{45} = \frac{\cancel{a}r^7}{\cancel{a}r^4}$

$8 = r^3$

$\boxed{r = 2}$

sub in ①

$a(2)^4 = 45$

$a = \frac{45}{16}$

$\therefore t_n = \frac{45}{16}(2)^{n-1}$

(ii) $t_{20} = \frac{45}{16}(2)^{19}$

$= \frac{45}{2^4}(2)^{19}$

$= 45(2)^{15}$

$= 1,474,560$

$\therefore$ The 20^{th} term is 1,474,560.

12. (i) $\frac{t_3}{t_2} = \frac{\cancel{11520}^{3}}{\cancel{7680}_{2}}$ ÷ 384

$= \frac{3}{2}$

$\frac{t_2}{t_1} = \frac{\cancel{7680}^{3}}{\cancel{5120}_{2}}$ ÷ 256

$= \frac{3}{2}$

$\because r = \frac{3}{2}$

$\therefore$ geometric and $t_n = 5120\left(\frac{3}{2}\right)^{n-1}$

(ii) $t_9 = 5120\left(\frac{3}{2}\right)^8$

$= \cancel{5120}^{20}\left(\frac{6561}{\cancel{256}_{1}}\right)$

$= 131\,220$

$\therefore$ If the pattern continues there will be 131 220 bacteria by the 9^{th} observation.