U7/L6 Review

1. $A = P(1+i)^n$

$= 21500\left(1+\frac{0.041}{12}\right)^{36}$

$\doteq 24\,308.92$

∴ The inheritance will be worth \$24 308.92.

2. $FV = \frac{R[(1+i)^n - 1]}{i}$

$= \frac{25\left[\left(1+\frac{0.0375}{52}\right)^{520} - 1\right]}{\frac{0.0375}{52}}$

$\doteq 15\,766.22$

∴ Shannon's parents will be able to give her \$15 766.22.

3. a) $R = \frac{PVi}{[1-(1+i)^{-n}]}$

$= \frac{5000\,000\left(\frac{0.08}{12}\right)}{\left[1-\left(1+\frac{0.08}{12}\right)^{-840}\right]}$

$\doteq 33\,459.38$

∴ The monthly payment for 70 years will be \$33 459.38.

b) Interest Earned $= 33\,459.38(840) - 5\,000\,000$

$= \$23\,105\,879.20$

4. $PV = \frac{R[1-(1+i)^{-n}]}{i}$

$= \frac{275[1-(1.00625)^{-60}]}{0.00625}$

$\doteq 13\,723.96$

∴ The max price of a vehicle that Jon can afford is \$13 723.96

5. Text p.535 11c, 13, 14d, 15, 18, 20

11.c) (i) $FV = \frac{R[(1+i)^n - 1]}{i}$

$= \frac{2500[(1.01075)^{12} - 1]}{0.01075}$

$= \$31\,838.87$

(ii) Int. Earned $= 31\,838.87 - 2500(12)$

$= \$1\,838.87$

13. $R = \frac{FVi}{[(1+i)^n - 1]}$

$= \frac{25000(0.0075)}{[(1.0075)^{72} - 1]}$

$\doteq 263.14$

∴ Ernie needs to invest \$263.14/mth for 6yrs to have \$25 000.

14.d)
(i) $PV = \frac{R[1-(1+i)^{-n}]}{i}$

$= \frac{105.27[1-(1.016)^{-18}]}{0.016}$

$\doteq \$1635.15$

(ii) Int. Charged $= 105.27(18) - 1635.15$

$= \$259.71$

p.535 15, 18, 20

15. a) $R = \frac{PVi}{[1-(1+i)^{-n}]}$

$= \frac{136\,000(0.0055)}{[1-(1.0055)^{-240}]}$

$\doteq 1022.00$

∴ Paul's monthly payments are \$1022.00 for 20 yrs.

b) Int Charged $= 1022(240) - 136\,000$

$= \$109\,280.49$

18. Ken

(i) $FV = \frac{R[(1+i)^n - 1]}{i}$

$= \frac{100[(1.0045)^{420} - 1]}{0.0045}$

$= 124\,252.52$

∴ At age 55, Ken will have \$124 252.52, after making monthly payments of \$100 for 35 years.

(ii) Adam

$R = \frac{FVi}{[(1+i)^n - 1]}$

$= \frac{124\,252.52(0.006)}{[(1.006)^{216} - 1]}$

$\doteq 282.34$

∴ Because he starts 17 yrs later, Adam's monthly payments must be \$282.34 if they want to have the same final amt. which is \$182.34 more/month than Ken.

20. $PV = \frac{R[1-(1+i)^{-n}]}{i}$

$= \frac{17.85[1-(1.0025)^{-130}]}{0.0025}$

$\doteq 1979.06$

∴ The selling price of the guitar is \$1979.06.

$\boxed{\#6}$ p.468 3, 4, 7-9, 13, 14abe, 15e, 18af, 19bc, 22

3. a) 58, 73, 88, ...

(i) $t_n = 58 + (n-1)(15)$

$= 15n + 43$

(ii) $t_1 = 58,\ t_n = t_{n-1} + 15,\ n > 1$

b) −49, −40, −31, ...

(i) $t_n = -49 + (n-1)(9)$

$= 9n - 58$

(ii) $t_1 = -49,\ t_n = t_{n-1} + 9,\ n > 1$

c) 81, 75, 69, ...

(i) $t_n = 81 + (n-1)(-6)$

$= -6n + 87$

(ii) $t_1 = 81,\ t_n = t_{n-1} - 6,\ n > 1$

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4. $t_7 = 465,\ t_{13} = 219,\ n = 100$

(i) $t_n = a + (n-1)d$

① $465 = a + 6d$ ② $219 = a + 12d$

② − ①

$$\begin{array}{r} a + 12d = 219 \\ -\quad a + 6d = 465 \\ \hline 6d = -246 \\ \boxed{d = -41} \end{array}$$

Sub in ①

$a + 6(-41) = 465$

$\boxed{a = 711}$

(ii) $t_{100} = 711 + 99(-41)$

$= -3348$

∴ The 100th term is −3348.

$\boxed{\#6}$ p. 468 7–9, 13, 14a,b,e, 15e, 18af, 19b, 22

7. a) 5, 15, 45, ...

(i) $\because r = 3$

$\therefore$ geometric

(ii) $t_n = ar^{n-1}$

$t_6 = 5(3)^5$

$= 1215$

b) 0, 3, 8, ...

neither geom. or arithmetic

c) 288, 14.4, 0.72, ...

(i) $\because r = \frac{1}{20}$

$\therefore$ geometric

(ii) $t_n = ar^{n-1}$

$t_6 = 288\left(\frac{1}{20}\right)^5$

$= \frac{288}{3\,200\,000}$

$= \frac{9}{100\,000}$

$= 0.000\,09$

d) 10, 50, 90, ...

(i) $\because d = -40$

$\therefore$ arithmetic

(ii) $t_6 = 10 + 5(-40)$

$= -190$

e) 19, 10, 1, ...

(i) $\because d = -9$

$\therefore$ arithmetic

(ii) $t_6 = 19 + 5(-9)$

$= -26$

f) 512, 384, 288, ...

(i) $\because r = \frac{3}{4}$

$\therefore$ geom.

(ii) $t_6 = 512\left(\frac{3}{4}\right)^5$

$= \cancel{512}^{1}\left(\frac{243}{\cancel{1024}_{2}}\right)$

$= \frac{243}{2}$ or 121.5

8. a) $t_1 = 7$, $r = -3$

(i) $t_1 = 7$, $t_n = -3t_{n-1}$, $n > 1$

(ii) $t_n = 7(-3)^{n-1}$

(iii) First 5 terms:

7, −21, 63, −189, 567

b) $a = 12$, $r = \frac{1}{2}$

(i) $t_1 = 12$, $t_n = \frac{1}{2}t_{n-1}$, $n > 1$

(ii) $t_n = 12\left(\frac{1}{2}\right)^{n-1}$

(iii) 12, 6, 3, $\frac{3}{2}$, $\frac{3}{4}$

c) $t_2 = 36$, $t_3 = 144$

$\therefore r = 4$ and $t_1 = 9$

(i) $t_1 = 9$, $t_n = 4t_{n-1}$, $n > 1$

(ii) $t_n = 9(4)^{n-1}$

(iii) 9, 36, 144, 576, 2304

9. a) $t_n = 4n + 5$

(i) $\because d = 4$

$\therefore$ arithmetic

(ii) First 5 terms:

9, 13, 17, 21, 25

b) $t_n = \dfrac{1}{7n-3}$

(i) neither

(ii) $\frac{1}{4}, \frac{1}{11}, \frac{1}{18}, \frac{1}{25}, \frac{1}{32}$

c) $t_n = n^2 - 1$

(i) neither

(ii) 0, 3, 8, 15, 24

d) $t_1 = -17$, $t_n = t_{n-1} + n - 1$, $n > 1$

(i) neither

(ii) −17, −16, −14, −11, −7

p.468 13, 14abe, 15e, 18af, 19bc, 22

13. $\frac{1}{2}, \frac{2}{5}, \frac{3}{8}, \frac{4}{11}, \ldots$ $\therefore t_n = \dfrac{n}{t_1+3(n-1)}$

$$\therefore t_{100} = \frac{100}{2+3(99)}$$
$$= \frac{100}{299}$$

14. a) $1+9+17+\ldots$

$$S_n = \frac{n[2a+(n-1)d]}{2}$$
$$S_{50} = \frac{\overset{25}{\cancel{50}}[2+49(8)]}{\cancel{2}_1}$$
$$= 25(394)$$
$$= 9850$$

b) $21+17+13+\ldots$

$$S_n = \frac{n[2a+(n-1)d]}{2}$$
$$S_{50} = \frac{\overset{25}{\cancel{50}}[2(21)+49(-4)]}{\cancel{2}_1}$$
$$= 25(42-196)$$
$$= 25(-154)$$
$$= -3850$$

e) $17.5+18.9+20.3+\ldots$

$$S_{50} = \frac{\overset{25}{\cancel{50}}[2(17.5)+49(1.4)]}{\cancel{2}_1}$$
$$= 25(35+68.6)$$
$$= 25(103.6)$$
$$= 2590$$

15. e) $t_{12}=19$, $d=-4$, $n=25$

(i) $t_n = a+(n-1)d$

$$t_{12} = a+(11)(-4)$$
$$19 = a-44$$
$$a = 63$$

(ii) $S_n = \dfrac{n[2a+(n-1)d]}{2}$

$$= \frac{25[2(63)+24(-4)]}{2}$$
$$= \frac{25[126-96]}{2}$$
$$= \frac{25(\overset{15}{\cancel{30}})}{\cancel{2}_1}$$
$$= 375$$

18. a) $11+33+99+\ldots$ $a=11$, $r=3$

(i) $t_n = ar^{n-1}$

$$t_6 = (11)(3)^5$$
$$= 2673$$

and $t_7 = 2673(3)$
$$= 8019$$

(ii) $S_n = \dfrac{t_{n+1}-t_1}{r-1}$

$$S_6 = \frac{t_7-t_1}{3-1}$$
$$= \frac{8019-11}{2}$$
$$= 4004$$

f) $\frac{1}{2}+\frac{3}{10}+\frac{9}{50}+\ldots$ $a=\frac{1}{2}$, $r=\frac{3}{5}$

(i) $t_n = ar^{n-1}$

$$t_6 = \frac{1}{2}\left(\frac{3}{5}\right)^5$$
$$= \frac{1}{2}\left(\frac{243}{3125}\right)$$
$$= \frac{243}{6250}$$

and $t_7 = \dfrac{243}{6250}\left(\dfrac{3}{5}\right)$
$$= \frac{729}{31250}$$

(ii) $S_n = \dfrac{t_{n+1}-t_1}{r-1}$

$$S_6 = \left(\frac{729}{31250}-\frac{1}{2}\right) \div \left(\frac{3}{5}-1\right)$$
$$= \frac{729-15625}{31250} \times -\frac{5}{2}$$
$$= \frac{-14896}{6250} \times \frac{-1}{2}$$
$$= \frac{7448}{6250}$$
$$= \frac{3724}{3125}$$

p.468 19 bc, 22

19. b) $t_1 = 42$, $t_9 = 2112$, $S_8 = ?$

(ii) $S_n = \frac{t_{n+1} - t_1}{r-1}$

(i) $t_n = ar^{n-1}$

$2112 = 42r^8$

$r = \sqrt[8]{\frac{2112}{42}}$

$S_8 = \frac{t_9 - t_1}{r-1}$

$= \frac{2112-42}{\left(\sqrt[8]{\frac{2112}{42}} - 1\right)}$

$\doteq 3276.087\,347$

c) $t_1 = 320$, $t_2 = 80$, $\therefore r = \frac{8\cancel{0}}{32\cancel{0}}$ or $\frac{1}{4}$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_8 = 320\left[\left(\frac{1}{4}\right)^8 - 1\right] \div \left[\frac{1}{4} - 1\right]$

$= 320\left[\frac{1}{65536} - \frac{65536}{65536}\right] \div \frac{-3}{4}$

$= 320\left(\frac{-65535}{65536}\right) \times \frac{-4}{3}$

$\doteq 426.660$

22. a) $7 + 14 + 28 + \ldots + 3584$; $r = 2$

(i) $t_n = ar^{n-1}$

$3584 = 7(2)^{n-1}$

$512 = 2^{n-1}$

$2^9 = 2^{n-1}$

$\therefore 9 = n-1$

$\boxed{n = 10}$

(ii) $S_n = \frac{a(r^n - 1)}{r-1}$

$S_{10} = \frac{7[(2)^{10} - 1]}{2-1}$

$= 7(1023)$

$= 7161$

b) $-3 - 6 - 12 - 24 - \ldots - 768$; $r = 2$

(i) $t_n = ar^{n-1}$

$-768 = -3(2)^{n-1}$

$256 = 2^{n-1}$

$2^8 = 2^{n-1}$

$\therefore \boxed{n = 9}$

(ii) $S_n = \frac{a(r^n - 1)}{r-1}$

$S_9 = \frac{(-3)[2^9 - 1]}{2-1}$

$= (-3)(511)$

$= -1533$

c) $1 + \frac{5}{2} + \frac{25}{4} + \ldots + \frac{15625}{64}$ $r = \frac{5}{2}$

(i) $t_n = ar^{n-1}$

$\frac{15625}{64} = 1\left(\frac{5}{2}\right)^{n-1}$

$\left(\frac{5}{2}\right)^6 = \left(\frac{5}{2}\right)^{n-1}$

$\therefore 6 = n-1$

$\boxed{n = 7}$

(ii) $S_n = \frac{a(r^n - 1)}{r-1}$

$S_7 = \frac{1\left[\left(\frac{5}{2}\right)^7 - 1\right]}{\left(\frac{5}{2} - 1\right)}$

$= \left[\frac{78125}{128} - \frac{128}{128}\right] \div \frac{3}{2}$

$= \frac{77997}{\cancel{128}\,64} \times \frac{\cancel{2}\,1}{3}$

$= \frac{25999}{64}$

e) $1000 + 1000(1.06) + 1000(1.06)^2 + \ldots + 1000(1.06)^{12}$

$n = 12$, $a = 1000$, $r = 1.06$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_{12} = \frac{1000[(1.06)^{12} - 1]}{(1.06 - 1)}$

$\doteq 16\,869.94$

22.d) $96\,000 - 48\,000 + 24\,000 - \ldots + 375$; $r = -\frac{1}{2}$; $t_{n+1} = 375\left(\frac{-1}{2}\right)$
$= -\frac{375}{2}$

$$S_n = \frac{t_{n+1} - t_1}{r-1}$$
$$= \left[-\frac{375}{2} - 96\,000\right] \div \left(-\frac{1}{2} - 1\right)$$
$$= \left[-\frac{375}{2} - \frac{192\,000}{2}\right] \div -\frac{3}{2}$$
$$= -\frac{192\,375}{\cancel{2}} \times \frac{\cancel{2}}{-3}$$
$$= 64\,125$$

Reviews #7

$2 + 7 + 12 + \ldots = 893$, $a = 2$, $d = 5$

(i) $t_n = a + (n-1)d$
$t_n = 2 + (n-1)(5)$
$t_n = 5n - 3$

(ii) $S_n = \frac{n(t_1 + t_n)}{2}$
$893 = \frac{n(2 + 5n - 3)}{2}$
$1786 = n(5n-1)$
$0 = 5n^2 - n - 1786$
$0 = (5n + 94)(n - 19)$
$n = 19$ or $\cancel{-\frac{94}{5}}$ inadmissible

$\therefore$ There are 19 terms in the series.

Check

(i) $t_n = 5n - 3$
$t_{19} = 5(19) - 3$
$= 92$

(ii) $S_n = \frac{n(t_1 + t_n)}{2}$
$S_{19} = \frac{19(2 + 92)}{2}$
$= \frac{19(\cancel{94})^{47}}{\cancel{2}_1}$
$= 893$ ✓