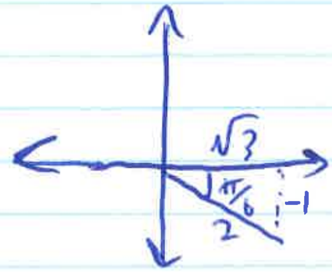
Dec 2019

Assignment 4 - Solutions

1.

a) $\sec \frac{11\pi}{6}$

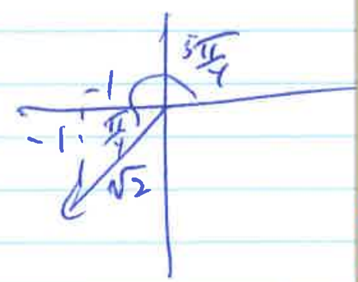


$= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

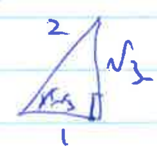
b) $\cos \frac{19\pi}{12}$

$= \cos\left(\frac{5\pi}{4} + \frac{\pi}{3}\right)$

$= \cos\frac{5\pi}{4}\cos\frac{\pi}{3} - \sin\frac{5\pi}{4}\sin\frac{\pi}{3}$



$= \frac{-1}{\sqrt{2}}\left(\frac{1}{2}\right) - \left(\frac{-1}{\sqrt{2}}\right)\frac{\sqrt{3}}{2}$



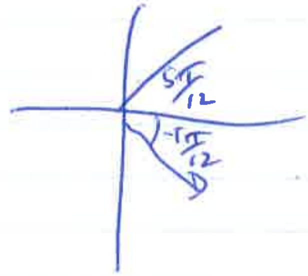
$= \frac{-1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{-1+\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$ (fine)

or $= \frac{\sqrt{6}-\sqrt{2}}{4}$

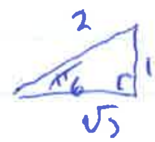
c) $\tan\left(-\frac{5\pi}{12}\right)$



$= -\tan\left(\frac{5\pi}{12}\right)$

$= -\tan\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right)$

$= -\tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$



$= -\left(\frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}}\right)$

$= -\left(\frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}(1)}\right)$

$= -\left(\frac{\sqrt{3}+3}{3} \div \frac{3-\sqrt{3}}{3}\right)$

$= -\left(\frac{\sqrt{3}+3}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}}\right)$

$= \left(\frac{3\sqrt{3}+3+9+3\sqrt{3}}{6}\right)$

$= \left(\frac{12+6\sqrt{3}}{6}\right) = -(2+\sqrt{3}) = -2-\sqrt{3}.$

2. a) $\sin\left(\frac{\pi}{4}-x\right)-\sin\left(\frac{5\pi}{4}+x\right)$

$= \sin\frac{\pi}{4}\cos x-\cos\frac{\pi}{4}\sin x-\left(\sin\frac{5\pi}{4}\cos x+\cos\frac{5\pi}{4}\sin x\right)$

$= \sin\frac{\pi}{4}\cos x-\cos\frac{\pi}{4}\sin x-\sin\frac{5\pi}{4}\cos x-\cos\frac{5\pi}{4}\sin x.$

$= \frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x-\left(-\frac{\sqrt{2}}{2}\cos x\right)-\left(-\frac{\sqrt{2}}{2}\sin x\right)$

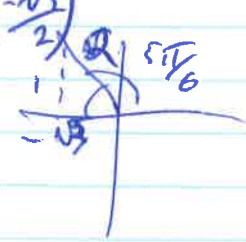
$= \frac{2\sqrt{2}}{2}\cos x+\frac{2\sqrt{2}}{2}\sin x$

$= \sqrt{2}\cos x+\sqrt{2}\sin x.$

b) $\cos\left(\theta+\frac{5\pi}{6}\right)+\sin\left(\theta-\frac{4\pi}{3}\right)$

$= \cos\theta\cos\frac{5\pi}{6}-\sin\theta\sin\frac{5\pi}{6}+\sin\theta\cos\frac{4\pi}{3}-\cos\theta\sin\frac{4\pi}{3}$

$= \cos\theta\left(-\frac{\sqrt{3}}{2}\right)-\sin\theta\left(\frac{1}{2}\right)+\sin\theta\left(-\frac{1}{2}\right)-\cos\theta\left(-\frac{\sqrt{3}}{2}\right)$



$= -\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta-\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta$

$= -\sin\theta.$

3.

a) $\sin(A-B)\sin(A+B) = \sin^2 A - \sin^2 B$

$LS = (\sin A\cos B - \cos A\sin B)(\sin A\cos B + \cos A\sin B)$

$= \sin^2 A\cos^2 B + \cancel{\sin A\cos B\cos A\sin B} - \cancel{\sin A\cos B\cos A\sin B} - \cos^2 A\sin^2 B$

$= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B.$

$= \sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B$

$= \sin^2 A - \sin^2 B$

$= RS.$ $\square$

b) $\tan x = \dfrac{\tan x - 1}{1 - \cot x}.$

$$RS = \frac{\frac{\sin x}{\cos x} - 1}{1 - \frac{\cos x}{\sin x}}$$

$$= \frac{\frac{\sin x - \cos x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}}$$

$$= \frac{\cancel{\sin x - \cos x}}{\cos x} \cdot \frac{\sin x}{\cancel{\sin x - \cos x}}$$

$$= \frac{\sin x}{\cos x}$$

$= \tan x$ $\square$

$= LS.$

c) $(\tan x + \sec x)^2 = \dfrac{\sin x + 1}{1 - \sin x}$

$$LS = \tan^2 x + 2\tan x \sec x + \sec^2 x$$

$$= \tan^2 x + 2\tan x \sec x + 1 + \tan^2 x$$

$$= 2\tan^2 x + 2\tan x \sec x + 1$$

$$= 2\frac{\sin^2 x}{\cos^2 x} + 2\frac{\sin x}{\cos x}\frac{1}{\cos x} + 1$$

$$= \frac{2\sin^2 x + 2\sin x}{\cos^2 x} + 1$$

$$= \frac{2\sin^2 x + 2\sin x + \cos^2 x}{\cos^2 x}$$

$$= \frac{2\sin^2 x + 2\sin x + 1 - \sin^2 x}{\cos^2 y}$$

$$= \frac{\sin^2 x + 2\sin x + 1}{1 - \sin^2 x}$$

$$= \frac{(\sin x + 1)^2}{(1 - \sin x)(1 + \sin x)}$$

$= \sin x + 1$ □.

d) if $\log(\sec x+\tan x) + \log(\sec x-\tan x) = 0$

then $\log[(\sec x+\tan x)(\sec x-\tan x)] = 0$

$10^0 = (\sec x+\tan x)(\sec x-\tan x)$

and $1 = (\sec x+\tan x)(\sec x\cancel{+}\tan x)$

so if the equation is true we only need to prove above equation.

$(\sec x+\tan x)(\sec x-\tan x)$

$= \sec^2 x - \tan^2 x$

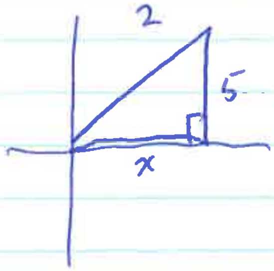
$= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$

$= \frac{1-\sin^2 x}{\cos^2 x}$

$= \frac{\cos^2 x}{\cos^2 x}$

$= 1.$ $\square$

4. a) $\sin x = \frac{2}{5}$



$5^2 = x^2 + 2^2$

$25 - 4 = x^2$

$x = \sqrt{21}$

$\sin 2x = 2 \sin x \cos x$

$= 2\left(\frac{2}{5}\right)\left(\frac{\sqrt{21}}{5}\right)$

$\sin 2x = \frac{4\sqrt{21}}{25}$

$\cos 2x = \cos^2 x - \sin^2 x$

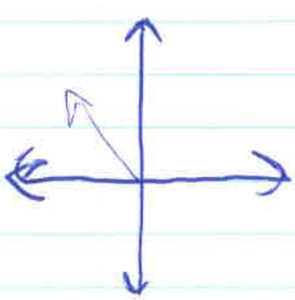
$= \left(\frac{\sqrt{21}}{5}\right)^2 - \left(\frac{2}{5}\right)^2$

$= \frac{21}{25} - \frac{4}{25}$

$\cos 2x = \frac{17}{25}$

b)

$\cos 2x = -\frac{7}{8}$



$\cos 2x = 2\cos^2 x - 1$

$\frac{-7}{8} = 2\cos^2 x - 1$

$-7 = 16\cos^2 x - 8$

$0 = 16\cos^2 x - 1$

$0 = (4\cos x - 1)(4\cos x + 1)$

$\cos x = \frac{1}{4}$ or $\cos x = -\frac{1}{4}$

which is it?

since $0 < x < \pi$

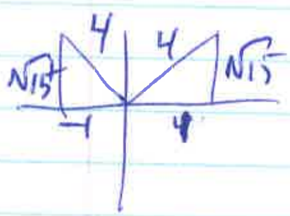
but $\cos 2x < 0 \rightarrow \frac{\pi}{2} < 2x < \frac{3\pi}{2}$

$$\frac{\pi}{4} < x < \frac{3\pi}{4}$$

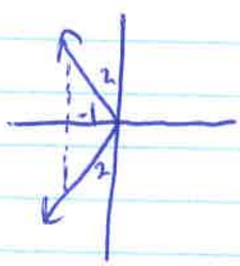
both values are possible. However $\sin x > 0$.

$\cos x = \pm 1/4$ $\qquad$ $\sin x = \frac{\sqrt{15}}{4}$

$16 = 1^2 + y^2$



5. a) $\cos x = -1/2$



$x = (\pi - \pi/3) + 2\pi k \quad k \in I$ or $x = (\pi + \pi/3) + 2\pi k, k \in I$

$$x = \frac{2\pi}{3} + 2\pi k, \; k \in I$$

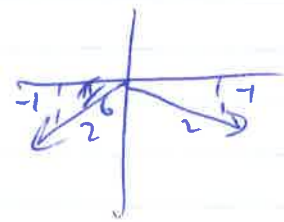
or $x = \frac{4\pi}{3} + 2\pi k, \; k \in I$

b) $\cos 2x = 1 + \sin x$

$1 - 2\sin^2 x = 1 + \sin x$

$2\sin^2 x + \sin x = 0$

$\sin x (2\sin x + 1) = 0$



$\sin x = 0$ or $2\sin x + 1 = 0$

$x = 0, \pi, 2\pi \ldots$ $\qquad \sin x = -\frac{1}{2}$

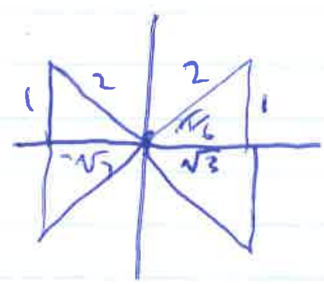
$\boxed{x = \pi k,\ k \in I}$

or

$\boxed{x = \frac{7\pi}{6} + 2\pi k,\ k \in I, \quad x = \frac{11\pi}{6} + 2\pi k,\ k \in I}$

c) $\cot^4 x - 2\cot^2 x - 3 = 0$

$(\cot^2 x + 1)(\cot^2 x - 3) = 0$



$\cot^2 x + 1 = 0$ or $\cot^2 x - 3 = 0$

$\cot^2 x = -1$ $\qquad \cot x = \pm\sqrt{3}$

not possible.

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \ldots\ldots$

$\boxed{x = \frac{\pi}{6} + \pi k,\ k \in I. \quad \text{or} \quad x = \frac{5\pi}{6} + \pi k,\ k \in I}$

d) $3\tan^3 x - \tan x = 0$

$\tan x\,(3\tan^2 x - 1) = 0$

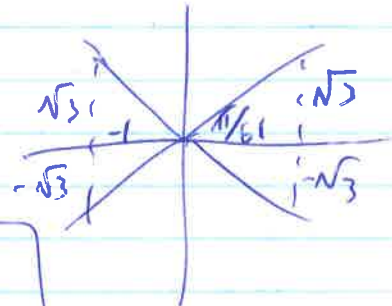
$0, \pi, 2\pi \ldots$ $\quad \tan x = 0$

$\boxed{x = \pi k, \ k \in I}$

$3\tan^2 x - 1 = 0$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{1}{\sqrt{3}}$



$$\boxed{\begin{array}{l} x = \frac{\pi}{6} + \pi k, \ k \in I \\ x = \frac{5\pi}{6} + \pi k, \ k \in I \end{array}}$$

e) $\sin\left(x + \frac{\pi}{4}\right) = \sqrt{2}\cos x$

$\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} = \sqrt{2}\cos x$

$\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x = \sqrt{2}\cos x$

$\frac{1}{2}\sin x + \frac{1}{2}\cos x = \cos x$

$\frac{1}{2}\sin x = \frac{1}{2}\cos x$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$\boxed{x = \frac{\pi}{4} + \pi k, \ k \in I}$

6. $\cos 3x + \cos 2x + \cos x = 0$

$\cancel{\cos 2x} +$

$\cos 3x = \cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$

$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x$

$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$

$\therefore \cos 3x + \cos 2x + \cos x = 0$

$2\cos^3 x - \cos x - 2\sin^2 x \cos x + \cos 2x + \cos x = 0$

$2\cos^3 x - \cos x - 2\sin^2 x \cos x + (2\cos^2 x - 1) + \cos x = 0$

$2\cos^3 x \cancel{- \cos x} - 2(1 - \cos^2 x)\cos x + 2\cos^2 x - 1 \cancel{+ \cos x} = 0$

$2\cos^3 x - 2\cos x + 2\cos^3 x + 2\cos^2 x - 1 = 0$

$4\cos^3 x + 2\cos^2 x - 2\cos x - 1 = 0$

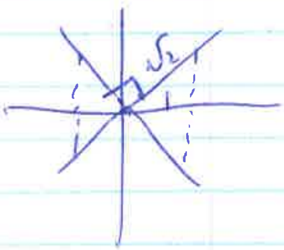
$2\cos^2 x(2\cos x + 1) - 1(2\cos x + 1) = 0$

$(2\cos^2 x - 1)(2\cos x + 1) = 0$

$2\cos^2 x - 1 = 0$ or $2\cos x + 1 = 0$

$\cos x = \pm \frac{1}{\sqrt{2}}$ $\cos x = -\frac{1}{2}$

$\cos x = \pm \frac{1}{\sqrt{2}}$

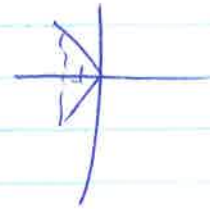


$x = \frac{\pi}{4} + \frac{\pi}{2}k, \ k \in I$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} + 2\pi k, \ k \in I$

$x = \frac{4\pi}{3} + 2\pi k, \ k \in I$

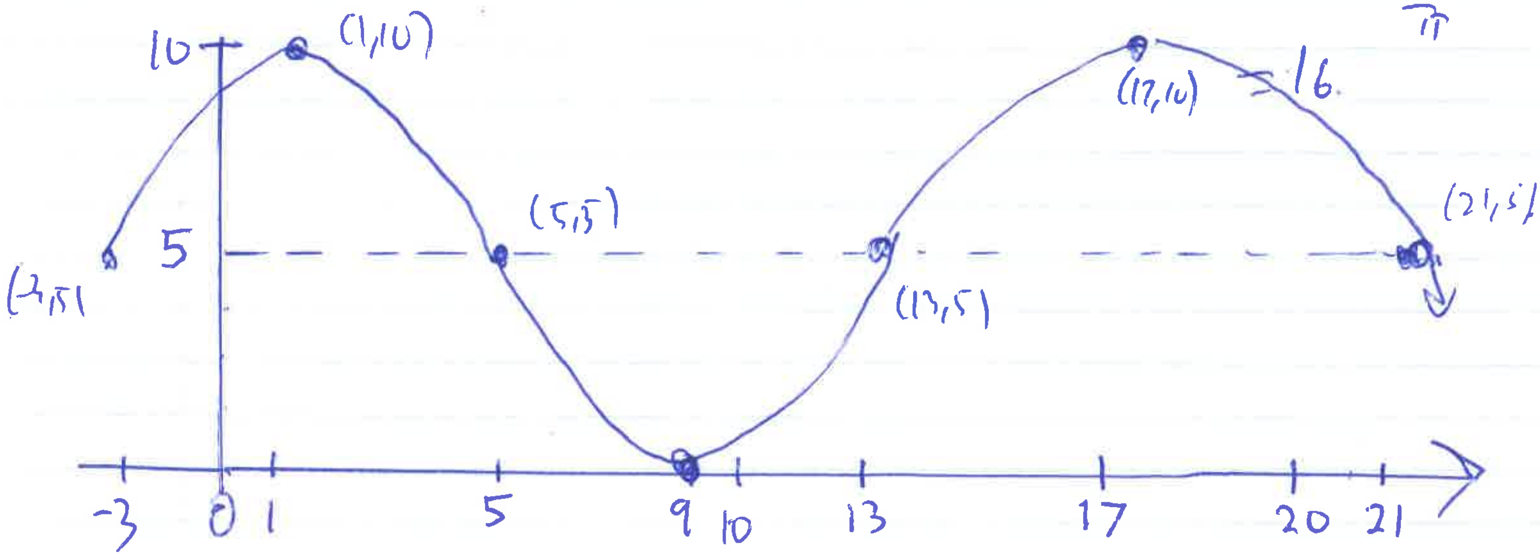


7. a) $y = 5\cos\left[\frac{\pi}{8}(x-1)\right]+5$

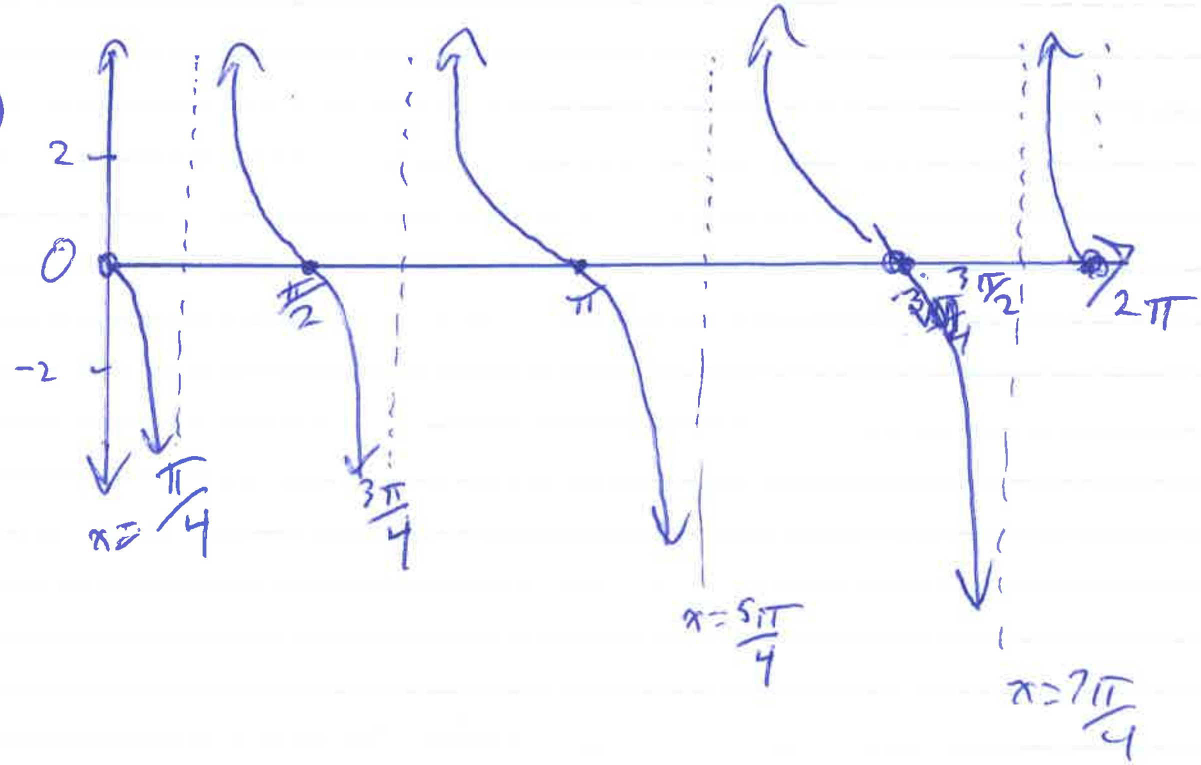
per. $= 2\pi \div \frac{\pi}{8}$

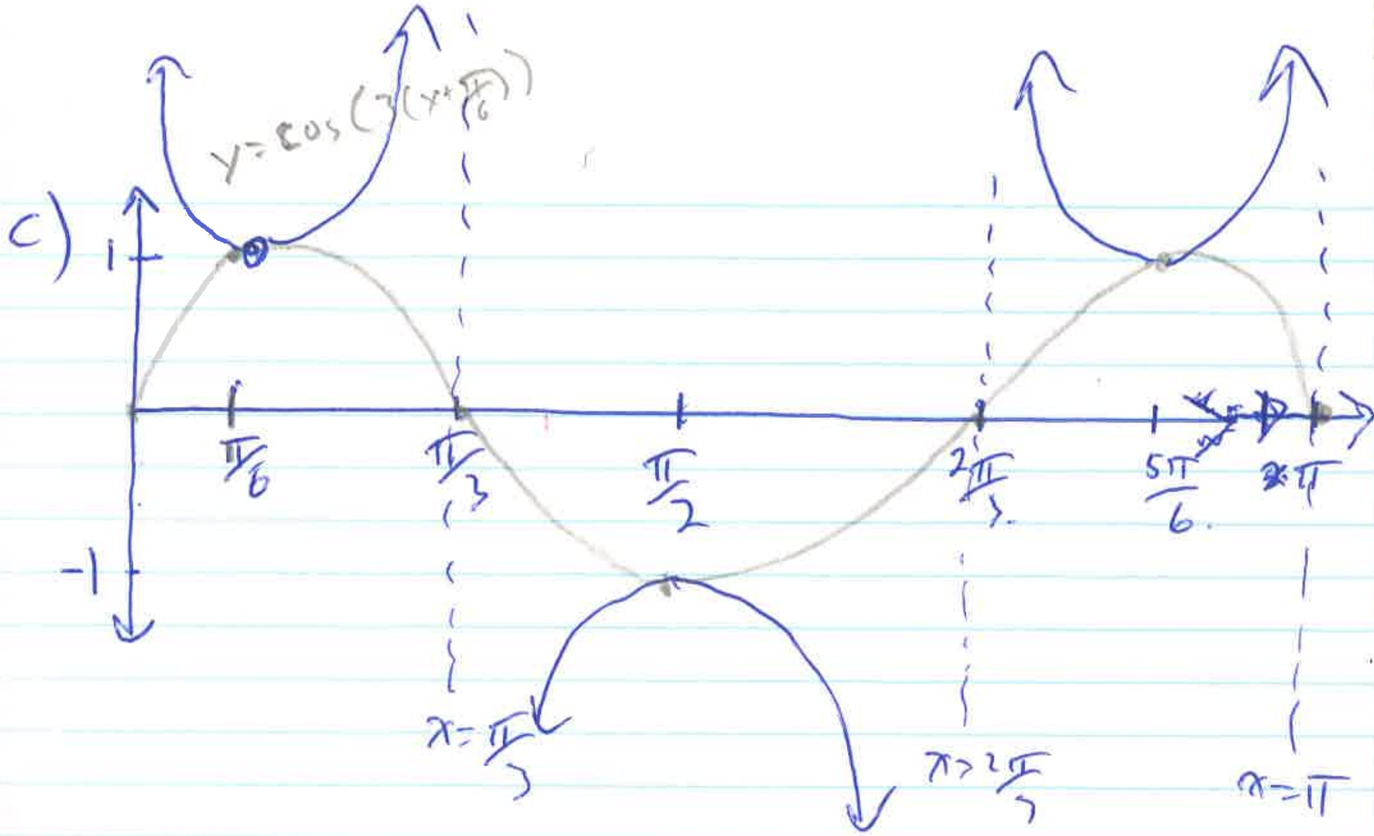
$= 2\pi \times \frac{8}{\pi}$

$= 16.$



b)





$\frac{2\pi}{3}$

2π

7. $y = \cos x$

$y = \sin 2x$

$\cos x = \sin 2x$

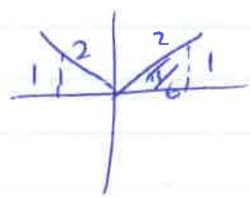
$\cos x = 2\sin x \cos x$

$2\sin x \cos x - \cos x = 0$

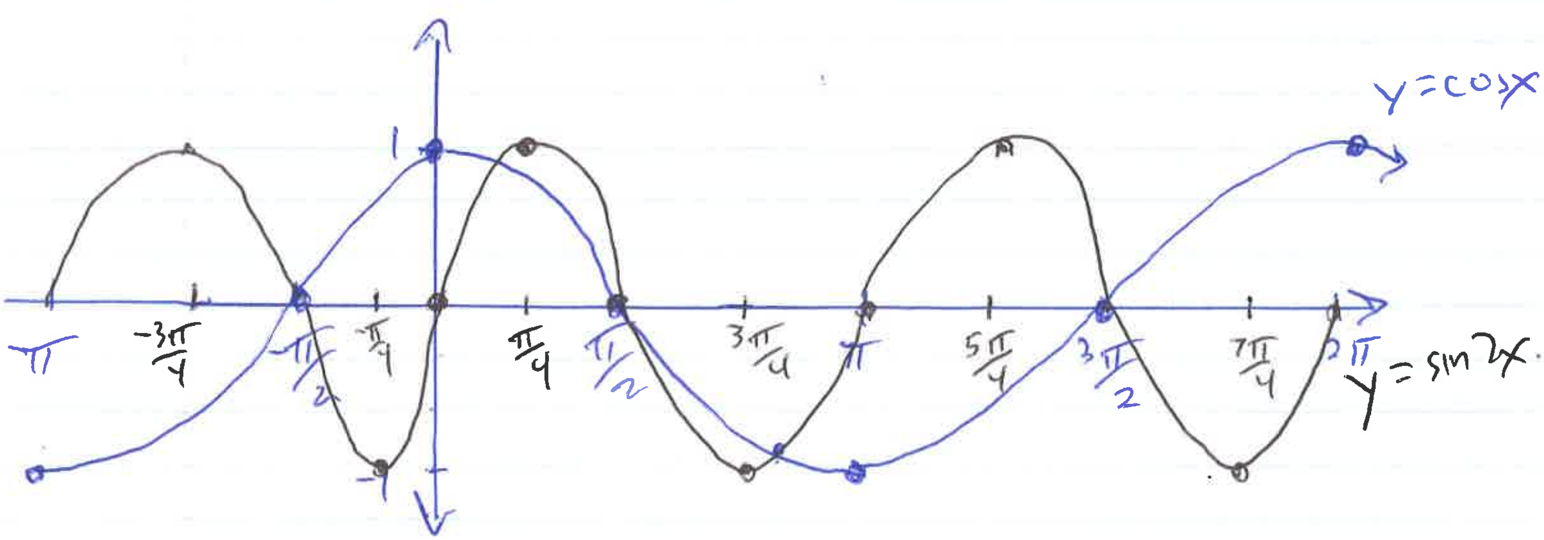
$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\qquad$ $2\sin x - 1 = 0$

$\sin x = 1/2$



$x = -\pi/2,\ \pi/2,\ 3\pi/2$ $\qquad$ $x = \pi/6,\ 5\pi/6.$



Points of intersection

$(-\pi/2, 0)$, $(\pi/6, \sqrt{3}/2)$, $(\pi/2, 0)$ $(5\pi/6, -\sqrt{3}/2)$ $(3\pi/2, 0)$