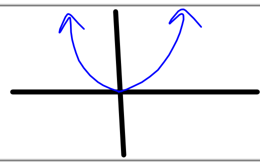
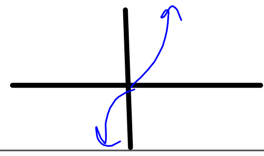
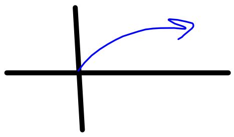
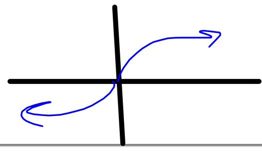
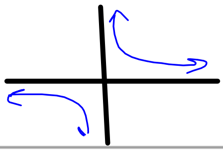
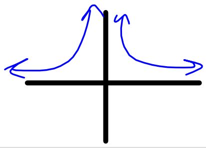
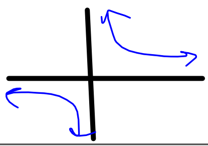
Introduction to Rational Functions

Even Function – A function where $f(x) = f(-x)$ for all *x* in the domain of *f(x)*.

Odd Function – A function where $f(-x) = -f(x)$ for all *x* in the domain of *f(x)*.

We will start by reviewing what the "classic curves" look like.

Equation	Critical Points	Asymptotes	Even or Odd?	Sketch
$y = x^2$	(0, 0)	/	even	
$y = x^3$	(0, 0)	/	odd	
$y = \sqrt{x}$	(0, 0)	/	N/A	
$y = \sqrt[3]{x}$	(0, 0)	/	odd	
$y = \frac{1}{x}$	none	$x = 0$ $y = 0$	odd	
$y = \frac{1}{x^2}$	none	$x = 0$ $y = 0$	even	
$y = \frac{1}{x^3}$	none	$x = 0$ $y = 0$	odd	

Rational Functions

We will spend a lot of time this unit exploring graphs of rational functions. A rational function is a function that has the form $f(x) = \frac{g(x)}{h(x)}$ where *g(x)* and *h(x)* are polynomials and $h(x) \neq 0$.

**Note in the case where h(x) = k (where k is a constant) really means that *f(x)* reduces to a polynomial function. **

Reciprocal Functions

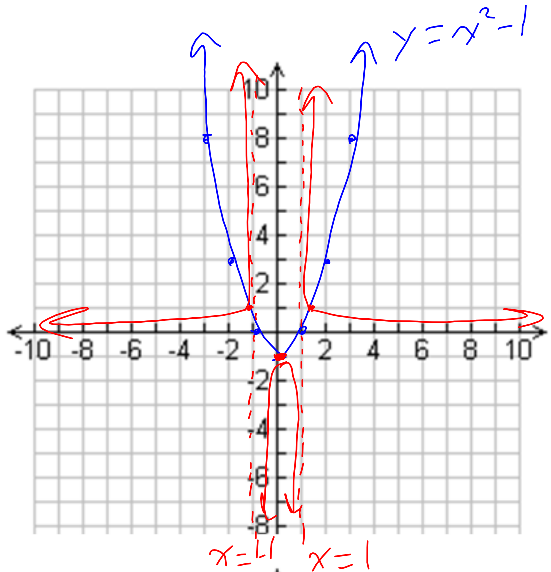
If $f(x) = \frac{g(x)}{h(x)}$ and *g(x) = 1,* then $f(x) = \frac{1}{h(x)}$ and *f(x)* can be treated as the reciprocal function of h(x). We have already looked at graphing functions of this form and we will review this concept.

Examples

$f(x) = \frac{1}{x^2 - 1}$

even

$f(x) = \frac{1}{(x-1)(x+1)}$

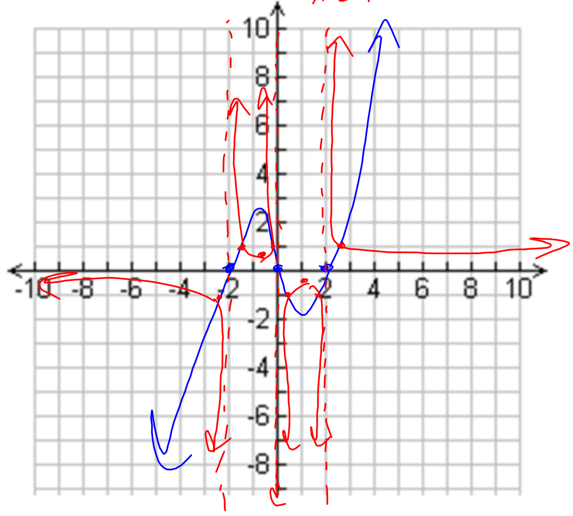


$f(x) = \frac{1}{x^3 - 4x}$

$y = x^3 - 4x$

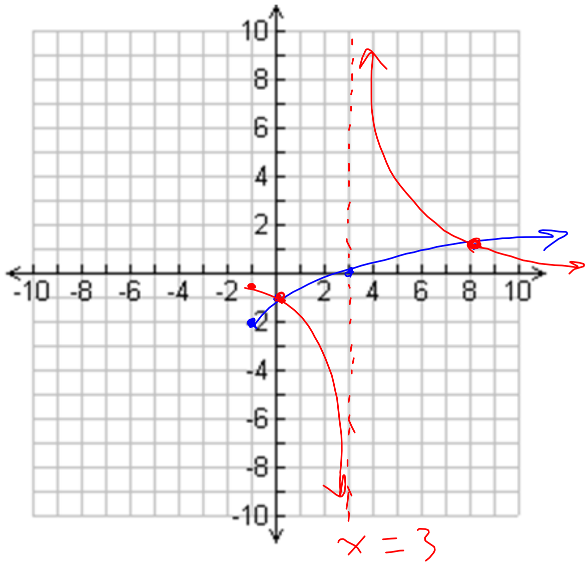
$y = x(x^2 - 4)$ *

$y = x(x-2)(x+2)$



$f(x) = \frac{1}{\sqrt{x+1}-2}$

$y = \sqrt{x+1} - 2$



Problem Set

1. Examine each equation below. Classify the function as **even, odd** or **neither.**

a) $f(x) = x^2 - 3x^4$ even

b) $y = -\sqrt[5]{x^2}$ even

c) $y = \frac{1}{x^2+2x}$ neither

d) $g(x) = \sin x$ odd

e) $y = \cos x$ even

f) $f(x) = \frac{2x}{3x^2-5}$ odd

g) y = 2x + 3 neither

2. Graph each function *f(x)*. Then graph the reciprocal function $y = \frac{1}{f(x)}$.

a) $f(x) = |x| - 4$

b) $f(x) = 2x - 4$

c) $f(x) = x^2 + 1$

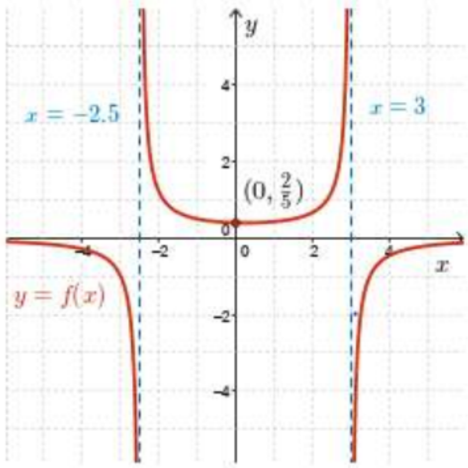
d) $f(x) = x^2 - 9$

e) $f(x) = log\, x$

3. Explain why the graph of $f(x) = \frac{a}{bx+c}$ must have a vertical asymptote. (a, b and c are real number constants).

4. Let $f(x) = \frac{a}{bx^2+cx+d}$ where a, b, c and d are real number constants. Under what conditions will *f(x)* have no vertical asymptotes? 1 vertical asymptote? 2 vertical asymptotes?

5. A graph is shown below. Find a possible equation for the graph in the form $f(x) = \frac{a}{bx^2+cx+d}$.



6. a) Sketch the graph of $f(x) = x^3 + 4x^2 + 4x$, by finding all intercepts and turning points. (great exam review).

   b) Use your answer to part a) to sketch the graph of $g(x) = \frac{1}{x^3+4x^2+4x}$.

ANSWERS

1. a) even b) even c) neither d) odd e) even f) odd g) neither
2. you will verify your answers tomorrow. 5. $y = \frac{-6}{2x^2-x-15}$ 6. Verify tomorrow.