



TEST SOLUTIONS

1.  $\theta = \frac{2\pi}{3}$

$C = r\theta$

$C = 18 \left(\frac{2\pi}{3}\right)$

$C = 12\pi \text{ cm.}$

2. a)  $\sin \frac{\pi}{2}$   
 $= 1$

b)  $\sec \frac{2\pi}{3}$   
 $= 2$

c)  $\tan \frac{5\pi}{6}$   
 $= \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

d)  $\cos \frac{4\pi}{3}$   
 $= -\frac{1}{2}$

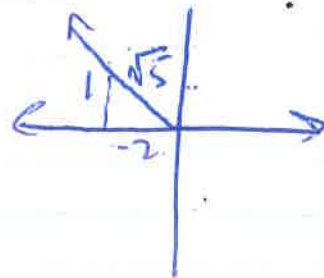
e)  $\csc \frac{7\pi}{4}$   
 $= -\sqrt{2}$

3.  $\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\frac{-2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2$

$= \frac{4}{5} - \frac{1}{5}$

$= \frac{3}{5}$



$\frac{2\pi}{3} \times \frac{1}{3}$

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$$4. \cos \frac{11\pi}{12}$$

$$= \cos \left( \frac{8\pi}{12} + \frac{3\pi}{12} \right)$$

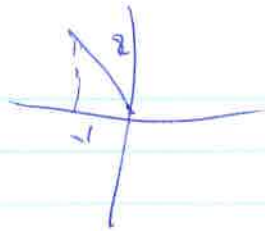
$$= \cos \left( \frac{2\pi}{3} + \frac{\pi}{4} \right)$$

$$= \cos \frac{2\pi}{3} \cos \frac{\pi}{4} - \sin \frac{2\pi}{3} \sin \frac{\pi}{4}$$

$$= \left( \frac{-1}{2} \right) \left( \frac{\sqrt{2}}{2} \right) - \left( \frac{\sqrt{3}}{2} \right) \left( \frac{\sqrt{2}}{2} \right)$$

$$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \frac{-\sqrt{2} - \sqrt{6}}{4}$$



$$5. a) \cos \theta = \cos \frac{6\pi}{7}$$

$$\theta = \pi + \frac{\pi}{7}$$

$$\boxed{\theta = \frac{8\pi}{7}}$$

$$b) \sin \frac{9\pi}{10} = \sin \theta$$

$$\boxed{\theta = \frac{\pi}{10}}$$

$$c) \cos \frac{\pi}{5} = \sin \theta$$

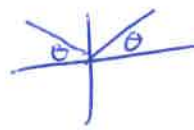
$$\frac{1}{2} = \frac{1}{5}$$

$$\frac{\pi}{5} = \frac{\pi}{2} - \theta$$

$$\theta = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10}$$

$\frac{\pi}{2}$

$$5c) \text{ or } \theta = \pi - \frac{3\pi}{10}$$
$$\boxed{\theta = \frac{7\pi}{10}}$$


$$\sin \theta = \sin(\pi - \theta)$$

6.

$$a) \text{ RS} = \frac{1}{\csc^2 x - 1} + 1$$

$$= \frac{1}{\frac{1}{\sin^2 x} - 1} + 1$$

$$= \frac{1}{\frac{1 - \sin^2 x}{\sin^2 x}} + 1$$

$$= \frac{\sin^2 x}{1 - \sin^2 x} + 1$$

$$= \frac{\sin^2 x}{\cos^2 x} + 1$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x \quad \square$$

$$b) \cot B - \cot A = \frac{\sin(A-B)}{\sin A \sin B}$$

$$RS = \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B}$$

$\Rightarrow$

$$LS = \frac{\cot B}{\sin B} - \frac{\cot A}{\sin A}$$

$$LS = \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B}$$

□

$$c) \sec 2x = \frac{\csc^2 x}{\csc^2 x - 2}$$

$$\begin{aligned} LS &= \sec 2x \\ &= \frac{1}{\cos 2x} \\ &= \frac{1}{\cos^2 x - \sin^2 x} \end{aligned}$$

$$= \frac{1}{\cancel{\cos^2 x} (1 - \sin^2 x) \sin^2 x}$$

$$= \frac{1}{1 - 2\sin^2 x}$$

$$RS = \frac{1}{\sin^2 x} = \left( \frac{1}{\sin^2 x} - 2 \right)$$

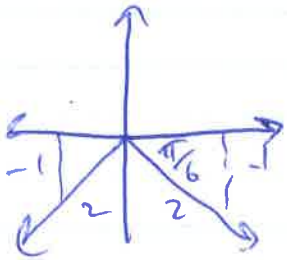
$$= \frac{1}{\sin^2 x} = \left( \frac{1 - 2\sin^2 x}{\sin^2 x} \right)$$

$$= \frac{1}{\sin^2 x} = \left( \frac{\sin^2 x}{1 - 2\sin^2 x} \right)$$

$$= \frac{1}{1 - 2\sin^2 x}$$

7. a)  $\sin x = -\frac{1}{2}$

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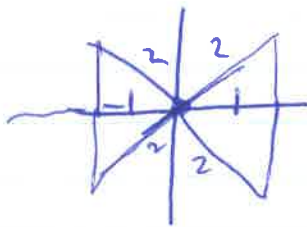
$$x = \frac{7\pi}{6} + 2\pi k, k \in \mathbb{I}$$

$$\text{or } x = \frac{11\pi}{6} + 2\pi k, k \in \mathbb{I}$$

b)  $\cos^2 x - 1 = 0$

$$\cos^2 x = 1$$

$$\cos x = \pm 1$$



$$\beta = \frac{\pi}{3}$$

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$$x = \frac{\pi}{3} + \pi k, k \in \mathbb{I}$$

$$\text{or } x = \frac{2\pi}{3} + \pi k, k \in \mathbb{I}$$

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$$c) \cos 2x = 2 - 3\sin x$$

$$1 - 2\sin^2 x = 2 - 3\sin x$$

$$0 = 2\sin^2 x - 3\sin x + 2 - 1$$

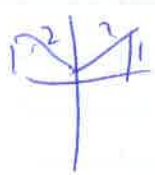
$$0 = 2\sin^2 x - 3\sin x + 1$$

$$0 = (2\sin x - 1)(\sin x - 1)$$

$$2\sin x - 1 = 0 \quad \text{or} \quad \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$$



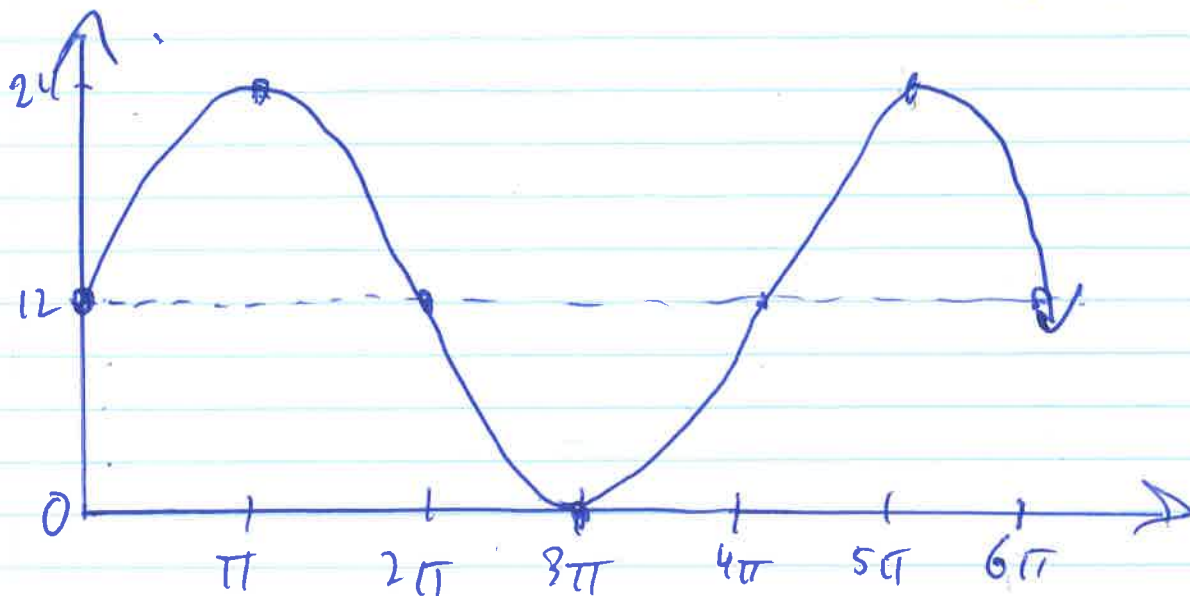
$$x = \frac{\pi}{6} + 2\pi k, k \in \mathbb{Z}$$

$$\text{or } x = \frac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}.$$

8.

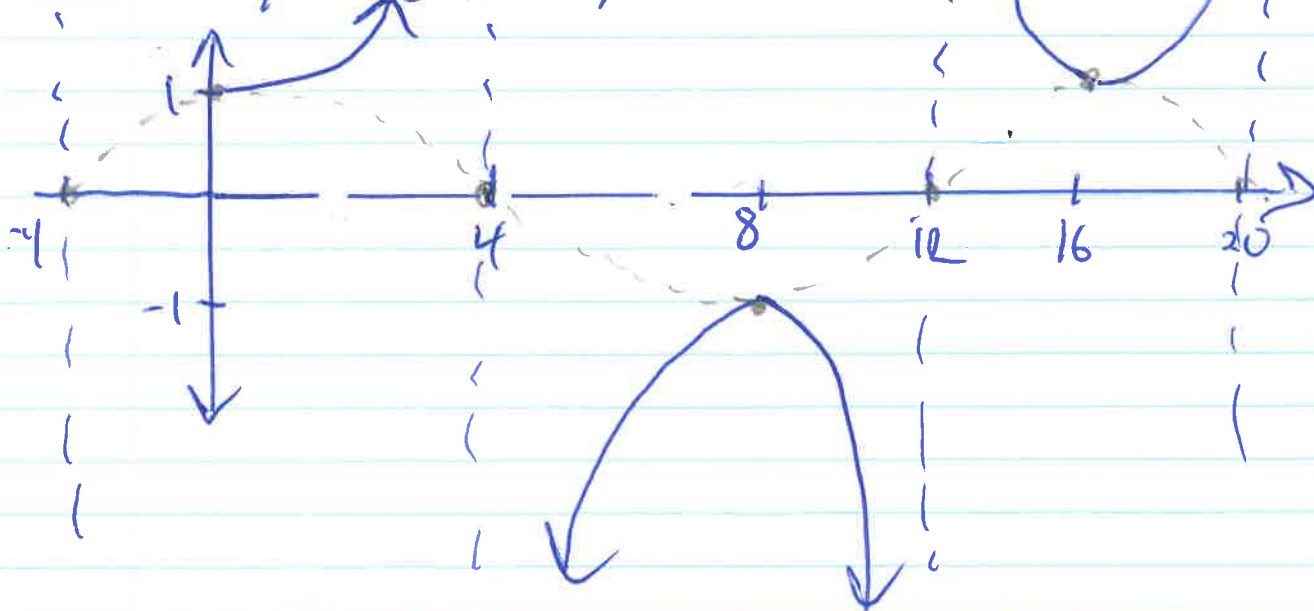
$$a) y = 12 \cos\left[\frac{1}{2}(x - \pi)\right] + 12$$

per. =  $4\pi$   
shift  $\pi$  right



$$b) y = \sec\left(\frac{\pi}{8}(x + 4)\right)$$

per. =  $2\pi \div \frac{\pi}{8} = 16$



$$9. a) \text{ per} = \frac{2\pi}{3} \quad y = -5\cos(3x)$$

$$b) \text{ per} = 40$$

$$40 = \frac{2\pi}{k}$$

$$40k = 2\pi$$

$$k = \frac{\pi}{20}$$

$$y = 10\cos\left(\frac{\pi}{20}t\right) + 20$$