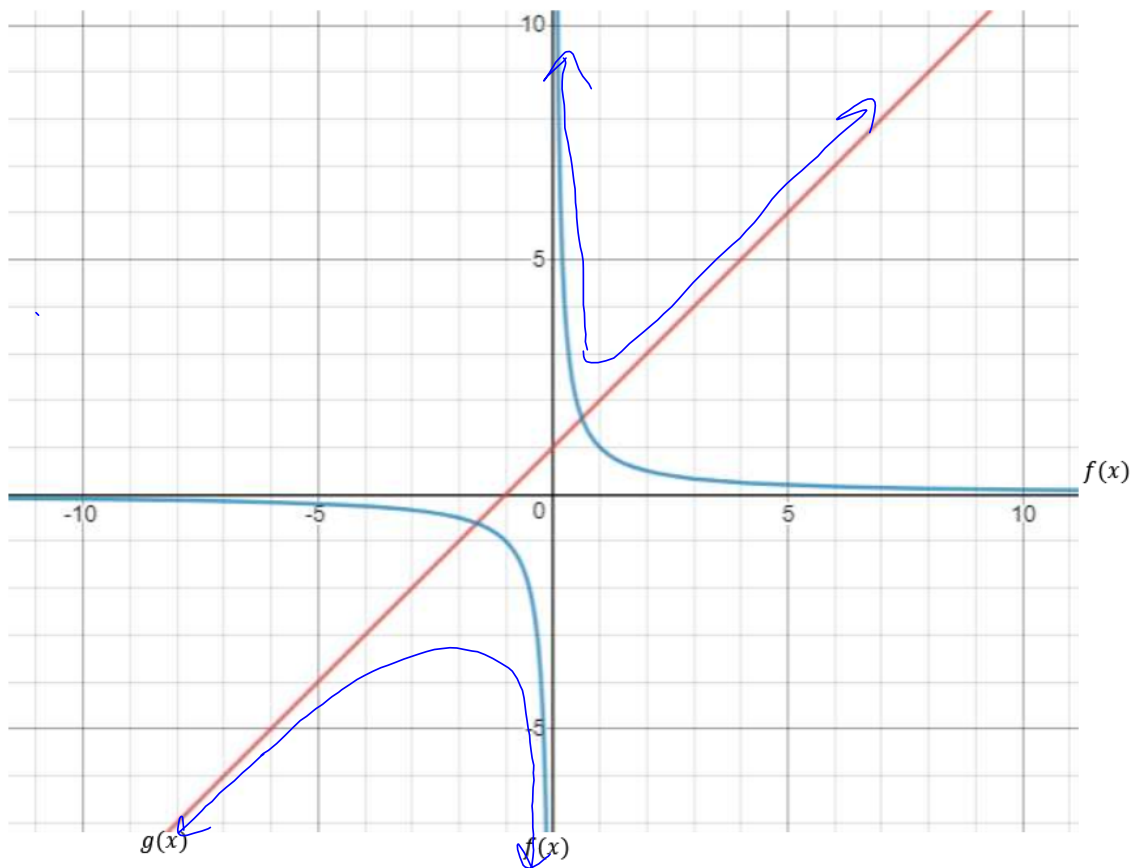


# Sums of Functions

We have been graphing rational functions by considering them as a **sum of functions**.

## Example 1

- a) The graphs of two functions  $f(x)$  and  $g(x)$  are shown the graph below. Let  $h(x) = f(x) + g(x)$  be a new function. Give a rough sketch of this function on the graph as well (use a different colour).



- b) In the graph above  $f(x) = \frac{1}{x}$  and  $g(x) = x + 1$ . What is the simplified equation for  $h(x)$ ?

$$h(x) = \frac{1}{x} + x + 1$$

$$h(x) = \frac{1}{x} + \frac{x(x+1)}{x}$$

$$h(x) = \frac{x^2 + x + 1}{x}$$

**Note:**  $f(x)$  is called a hyperbola. All of the functions belonging to this family are referred to as hyperbolas.

c) The graph of  $h(x)$  appears to have no intercepts. Can this be verified algebraically?

$$h(x) = \frac{x^2 + x + 1}{x}$$

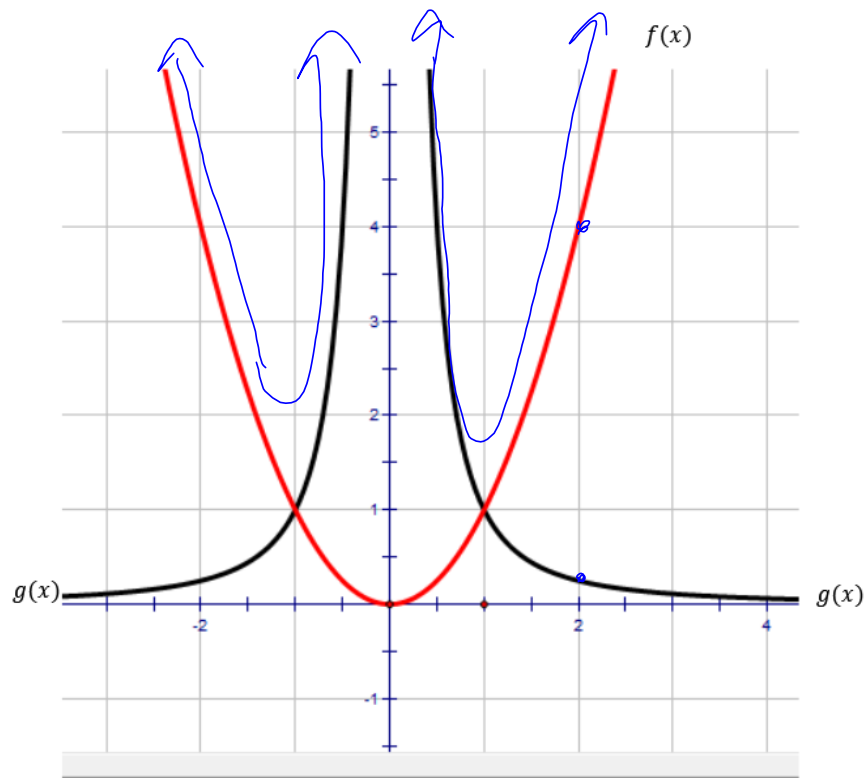
no  $x$ -int  $x^2 + x + 1 \neq 0$   
 $y$ -int  $x \neq 0$

d) What are the asymptotes for  $h(x)$ ? Can this be verified algebraically?

$x = 0$   
 $y = x + 1$  is oblique asymptote

Example #2

a) Two functions  $f(x)$  and  $g(x)$  are shown below. Let  $h(x) = f(x) + g(x)$ . Graph  $h(x)$  on the graph below as well.



b) Does  $h(x)$  appear to have any asymptotes?

V. A. at  $x = 0$       parabolic asymptote at  $y = x^2$

c) In the graph above  $f(x) = x^2$  and  $g(x) = \frac{1}{x^2}$ . Find a simplified equation for  $h(x)$ .

$$h(x) = x^2 + \frac{1}{x^2}$$

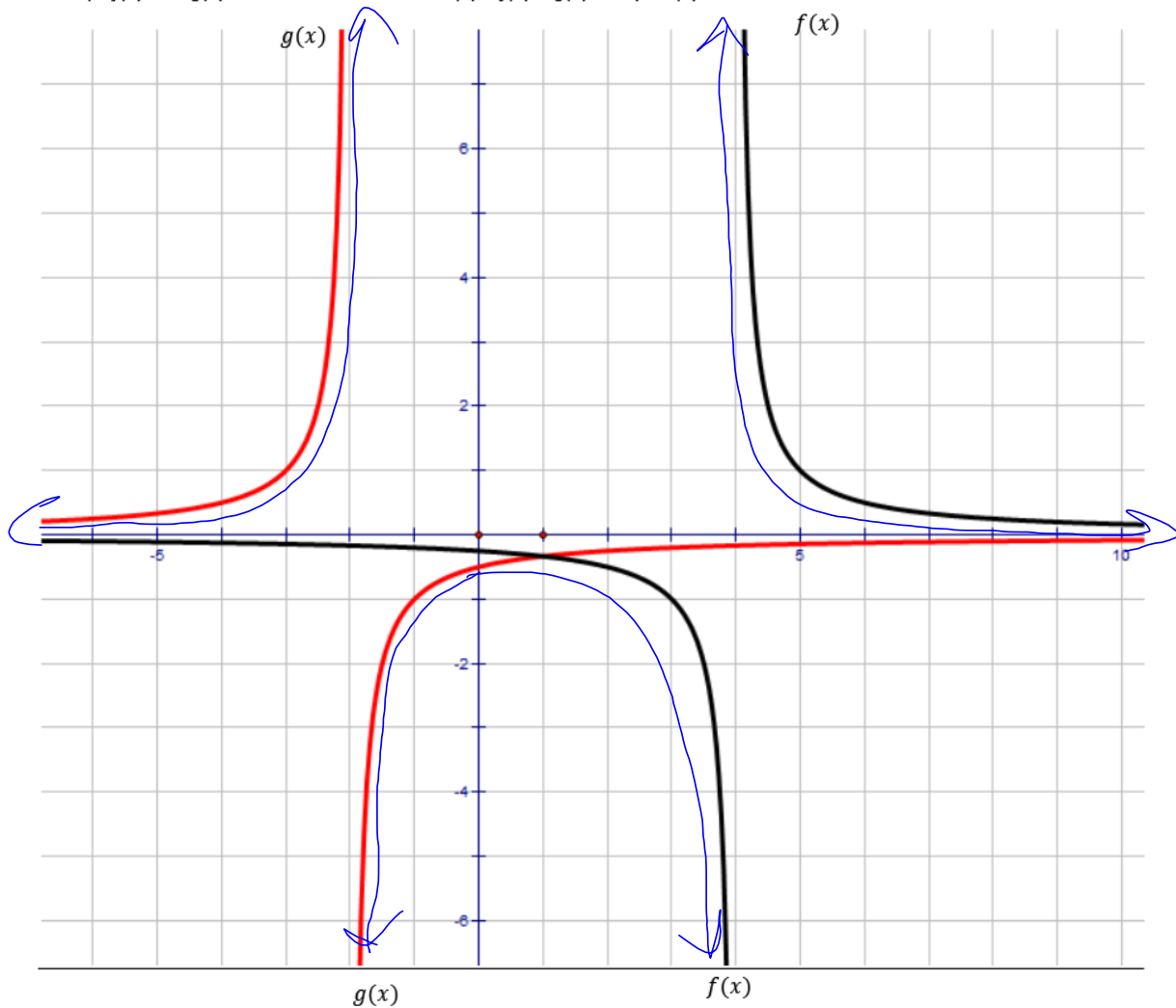
$$h(x) = \frac{x^4 + 1}{x^2}$$

d) Can the asymptotes for  $h(x)$  be verified algebraically?

yes

Example 3

a)  $f(x)$  and  $g(x)$  are shown below. Let  $h(x) = f(x) + g(x)$ . Graph  $h(x)$  below.



b) In the above example  $f(x) = \frac{1}{(x-4)}$  and  $g(x) = \frac{-1}{(x+2)}$ . Find a simplified expression for  $h(x)$ .

$$h(x) = \frac{1}{x-4} - \frac{1}{x+2} \qquad h(x) = \frac{6}{(x-4)(x+2)}$$

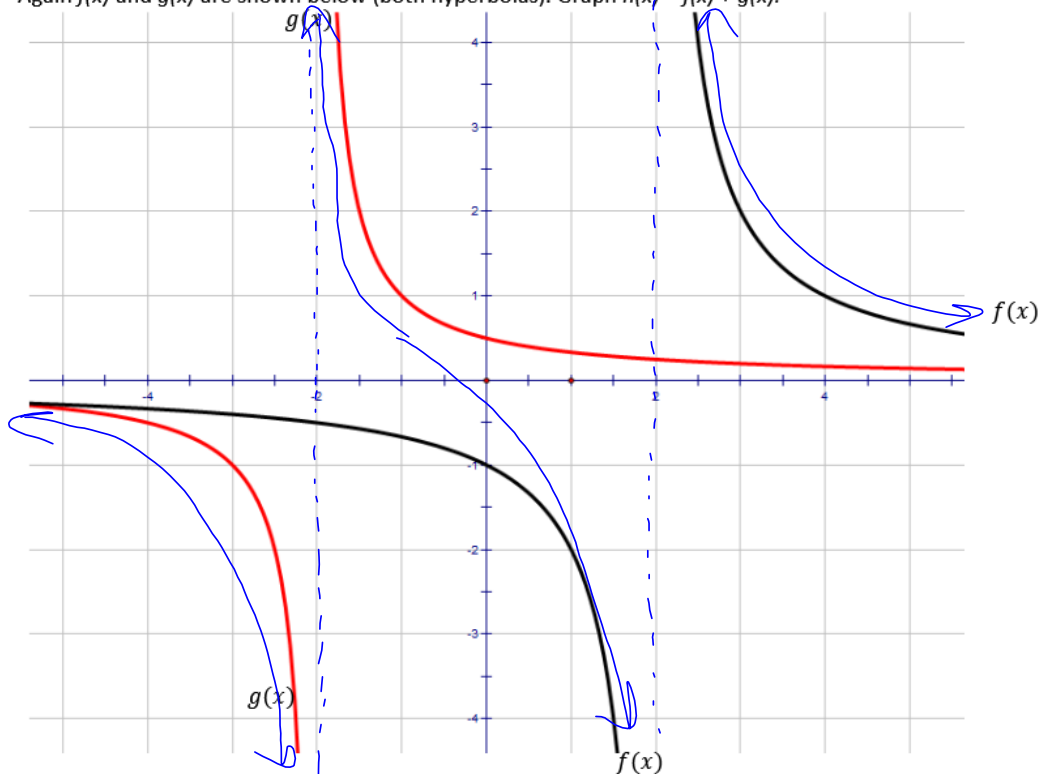
$$h(x) = \frac{x+2 - (x-4)}{(x-4)(x+2)}$$

c) Where are the asymptotes for  $h(x)$ ? Can they be verified algebraically?

V. A. at  $x = 4, x = -2$   
 H. A. at  $y = 0$

**Example 4**

a) Again  $f(x)$  and  $g(x)$  are shown below (both hyperbolas). Graph  $h(x) = f(x) + g(x)$ .



b) Here  $f(x) = \frac{2}{x-2}$  and  $g(x) = \frac{1}{x+2}$ . Find an equation for  $h(x)$ .

c) What are the asymptotes of  $h(x)$ ?

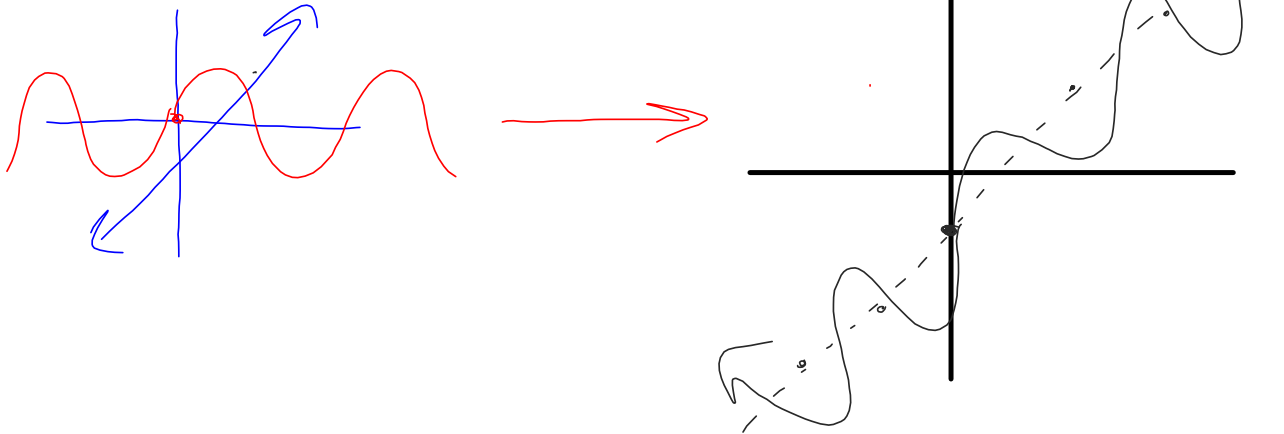
$$h(x) = \frac{2}{x-2} + \frac{1}{x+2}$$

$$h(x) = \frac{2(x+2) + x-2}{(x-2)(x+2)}$$

$$h(x) = \frac{3x+2}{(x-2)(x+2)}$$

**Example 5**

Sketch the graph of  $f(x) = \underline{x} - 3 + \underline{\sin x}$ , by considering  $f(x)$  as a sum of two functions.



**Homework**

Sketch the graphs of the following functions by expressing them as a sum of functions.

1)  $f(x) = \frac{x^2 - x - 20}{x + 3}$

2)  $f(x) = \frac{x^4 - x^2 + 1}{x^2}$

3)  $f(x) = \frac{x^4 + 1}{x}$