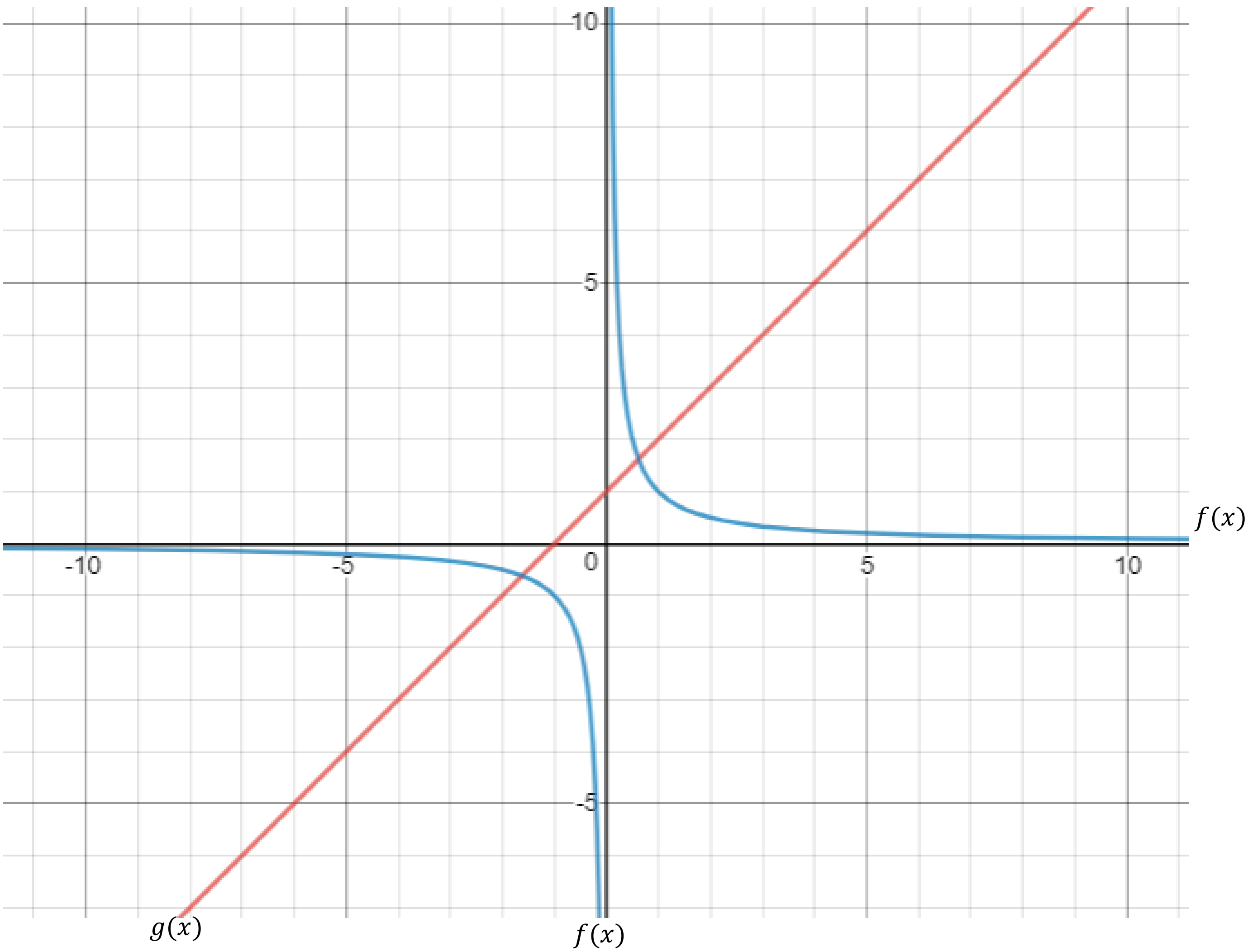
Sums of Functions

We have been graphing rational functions by considering them as a ***sum of functions.***

Example 1

a) The graphs of two functions *f(x)* and *g(x)* are shown the graph below. Let *h(x) = f(x) + g(x)* be a new function. Give a rough sketch of this function on the graph as well (use a different colour).



b) In the graph above $f(x) = \frac{1}{x}$ and $g(x) = x + 1$. What is the simplified equation for *h(x)?*

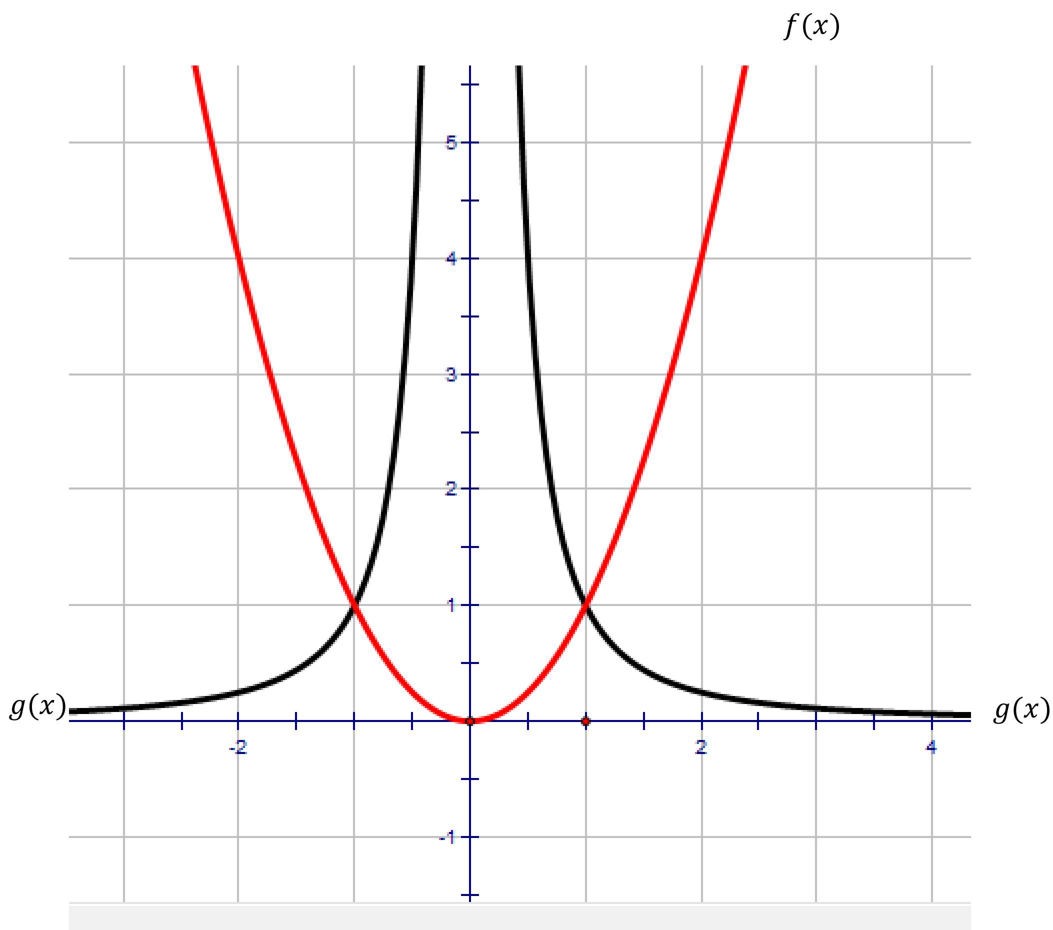
Note: *f(x)* is called a hyperbola. All of the functions belonging to this family are referred to as hyperbolas.

c) The graph of *h(x)* appears to have no intercepts. Can this be verified algebraically?

d) What are the asymptotes for *h(x)?* Can this be verified algebraically?

<u>Example #2</u>

a) Two functions *f(x)* and *g(x)* are shown below. Let *h(x) = f(x) + g(x).* Graph *h(x)* on the graph below as well.



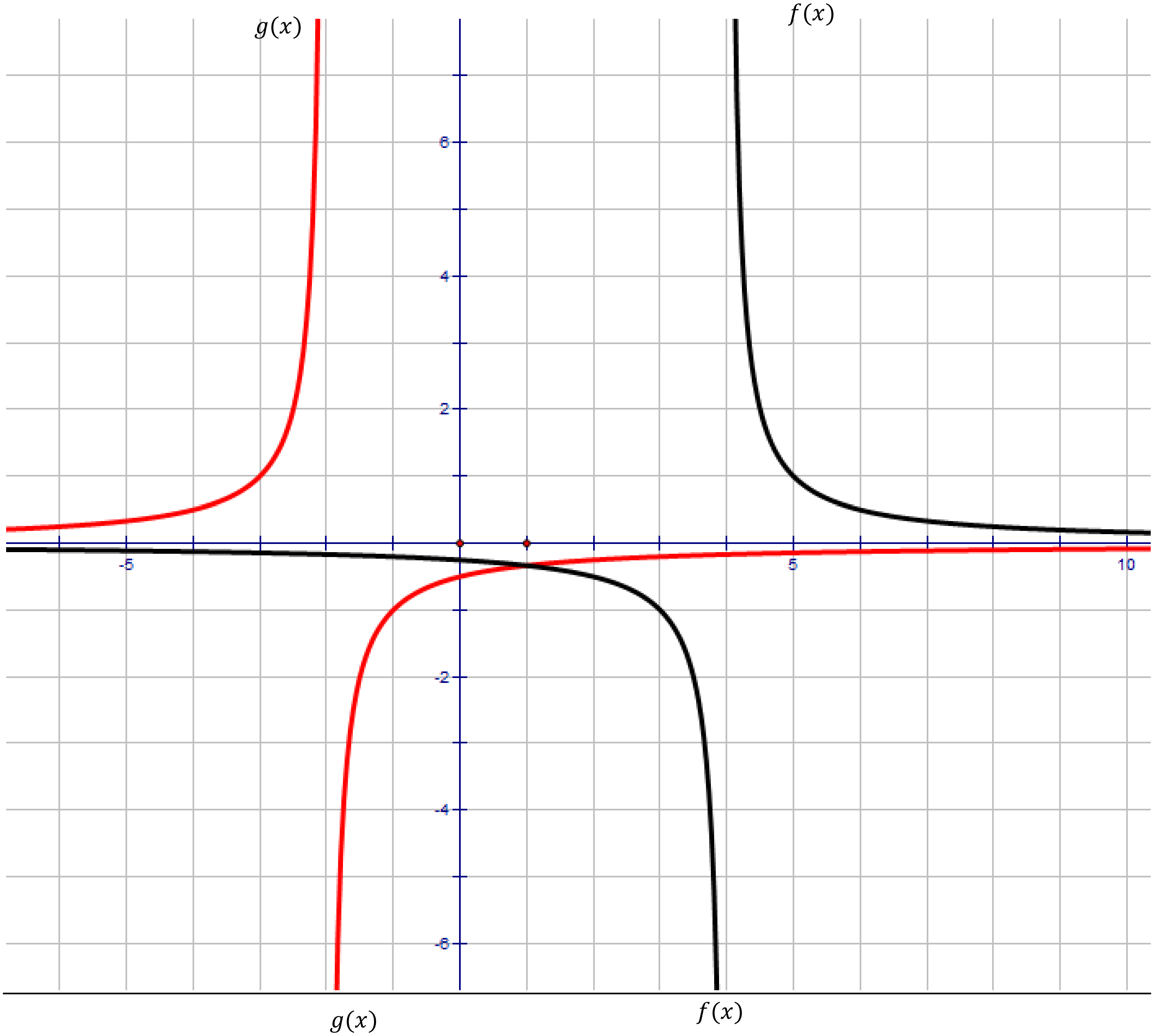
b) Does *h(x)* appear to have any asymptotes?

c) In the graph above $f(x) = x^2$ and $g(x) = \frac{1}{x^2}$. Find a simplified equation for *h(x).*

d) Can the asymptotes for *h(x)* be verified algebraically?

<u>Example 3</u>

a) *f(x)* and *g(x)* are shown below. Let *h(x) = f(x) + g(x).* Graph *h(x)* below.

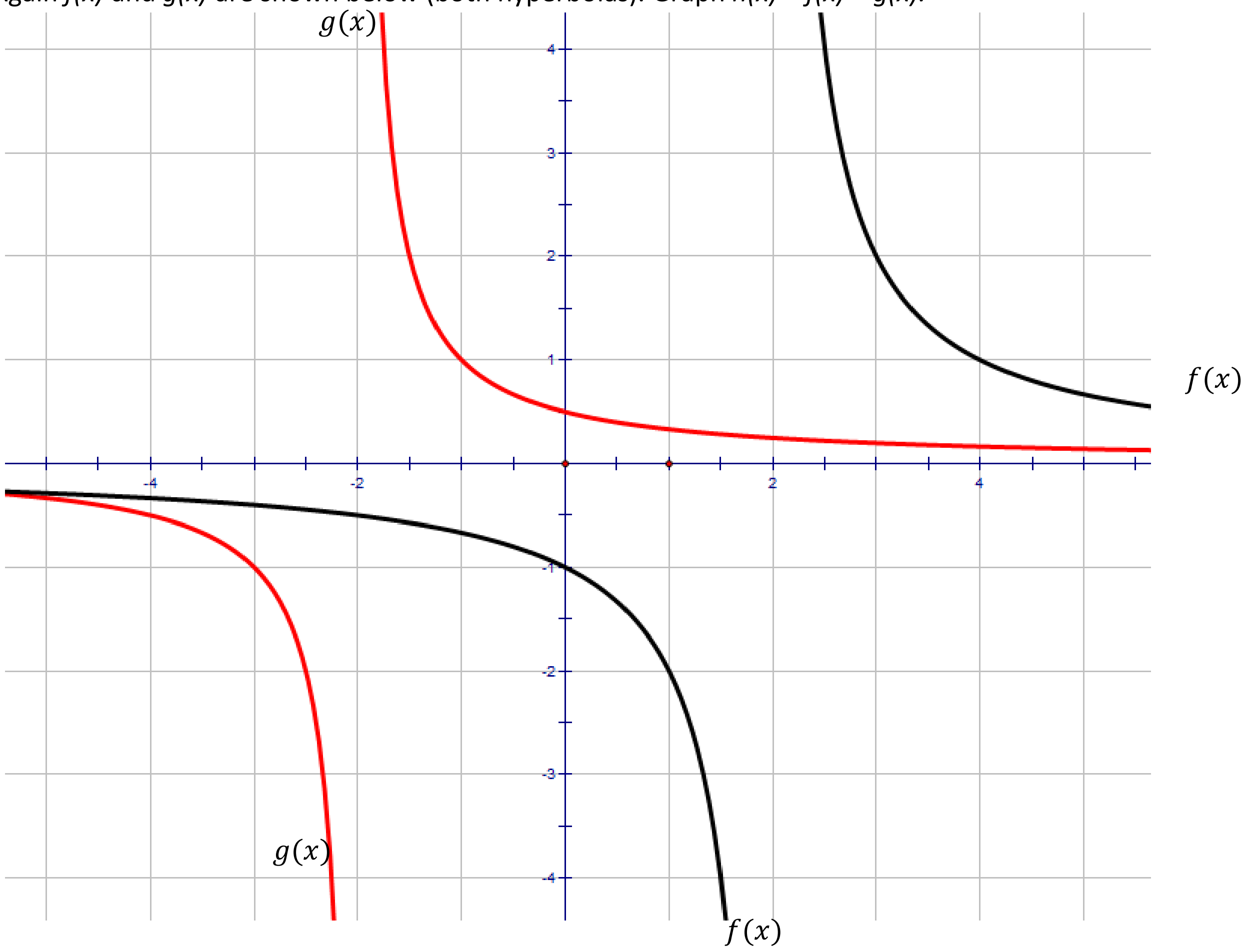


b) In the above example $f(x) = \frac{1}{(x-4)}$ and $g(x) = \frac{-1}{(x+2)}$. Find a simplified expression for *h(x).*

c) Where are the asymptotes for *h(x)?* Can they be verified algebraically?

<u>Example 4</u>

a) Again *f(x)* and *g(x)* are shown below (both hyperbolas). Graph *h(x) = f(x) + g(x).*



b) Here $f(x) = \frac{2}{x-2}$ and $g(x) = \frac{1}{x+2}$. Find an equation for *h(x).*

c) What are the asymptotes of *h(x)?*

Example 5

Sketch the graph of $f(x) = x - 3 + \sin x$, by considering *f(x)* as a sum of two functions.

Homework

Sketch the graphs of the following functions by expressing them as a sum of functions.

1) $f(x) = \frac{x^2 - x - 20}{x + 3}$

2) $f(x) = \frac{x^4 - x^2 + 1}{x^2}$

3) $f(x) = \frac{x^4 + 1}{x}$