Unit 4 Test: Exponents & Exponential Relationships

Name: _____

Marking Scheme:

Knowledge/Understanding: Questions #1 - 5
Application: Questions #7 - 10
TIPS: Question #6, 11
Communication: 1 mark per page (5 marks total)

78 72.

1. Evaluate each of the following. (Express as a rational number where necessary – no decimals).

a) $\left(\frac{1}{3}\right)^4$ b) 7^0 c) -6^2 d) $\left(\frac{2}{5}\right)^{-2}$ e) $(-2)^{-4}$

$= 3^{-4}$ $= 1$ $= -36$ $= \left(\frac{5}{2}\right)^2$ $= \frac{1}{(-2)^4}$

$= \frac{1}{81}$ $= \frac{25}{4}$ $= \frac{1}{16}$

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2. Use exponent laws to simplify each of the following (if possible). Then evaluate.

a) $\frac{(5^2)^3(5)^4(5)}{5^8}$ b) $\frac{6^{-1}}{6^2}$ c) $(3^2)^3 \div 3^3$

$= \frac{5^6(5^4)5}{5^8}$ ✓ $= 6^{-3}$ ✓ $= 3^6 \div 3^3$ ✓

$= \frac{5^{11}}{5^8}$ ✓ $= \frac{1}{6^3}$ ✓ $= 3^3$ ✓

$= 5^3 = 125$ $= \frac{1}{216}$ $= 27$ ✓

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d) $3^{-2} - 2^{-5} \times 2^3$ e) $\frac{2^{-3}}{2^5} \times \frac{2^{-2}}{2^{-3}}$ f) $3(2^{-2})$

$= 3^{-2} - 2^{-2}$ ✓ $= 2^{-8} \times 2$ ✓ $= 3\left(\frac{1}{4}\right)$ ✓

$= \frac{1}{9} - \frac{1}{4}$ ✓ $= 2^{-7}$ ✓ $= \frac{3}{4}$ ✓

$= \frac{4}{36} - \frac{9}{36} = -\frac{5}{36}$ ✓ $= \frac{1}{2^7}$ ✓

$= \frac{1}{128}$

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3. Simplify each of the following. Final answers should contain positive exponents only. You may need to expand first.

a) $2x^2y(3xy - 4y)$

$= 6x^3y^2 - 8x^2y^2$

b) $(2x)^5$

$= 32x^5$

c) $(2a^2b^{-1})^{-2}$

$= 2^{-2}a^{-4}b^2$

$= \frac{b^2}{4a^4}$

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d) $5a(2a^2 - b) + ab(4ab - 2)$

$= 10a^3 - 5ab + 4a^2b^2 - 2ab$

$= 10a^3 + 4a^2b^2 - 7ab$

e) $\frac{6m^2 - 12m^3}{3m}$

$= 2m - 4m^2$

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f) $\frac{(2xy^2)^3}{2xy^2}$

$= \frac{8x^3y^6}{2xy^2}$

$= 4x^2y^4$

g) $\frac{x^3(x^2)^{-3}(x)}{x^5}$

$= \frac{x^3x^{-6}x}{x^5}$

$= \frac{x^{-2}}{x^5}$

$= x^{-7}$

$= \frac{1}{x^7}$

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4. Convert each of the following numbers into scientific notation.

a) 235 000 000 $= 2.35 \times 10^{8}$

b) 0.0000023 $= 2.3 \times 10^{-6}$

c) $-345\,000$ $= -3.45 \times 10^{5}$

d) 530×10^{-6} $= 5.3 \times 10^{-4}$

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5. Evaluate the following by first converting each number to **scientific notation**. Express your final answer in **standard form**.

$$\frac{0.00000004 \times 50{,}000{,}000}{2{,}000{,}000}$$

$$= \frac{4 \times 10^{-8} \times 5 \times 10^{7}}{2 \times 10^{6}}$$

$$= \frac{20 \times 10^{-1}}{2 \times 10^{6}}$$

$$= 10 \times 10^{-7}$$

$$= 1 \times 10^{-6}$$

$$= 0.000001$$

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6. Every cubic metre of sand contains 1.4×10^{10} grains of sand. All of the sand from all of beaches in world has a total volume of 7.5×10^{11} cubic metres. How many grains of sand are there on all of the beaches of the world? (Answer in scientific notation).

$$1.4 \times 10^{10} \times 7.5 \times 10^{11}$$

$$= 10.5 \times 10^{21}$$

$$= 1.05 \times 10^{22}$$

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7. The population of a town is currently 32 000 people. The town increases in size at a rate of 2.5% per year.

a) Define an equation that models the population of the town.

$P = 32000(1.025)^n$

n = # of years
P = population.

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b) Use your equation to predict the size of the town in 7 years.

$P = 32000(1.025)^7$

$\doteq 38038$

8. One day Mr. Elliott notices 5 ants in his kitchen. Each day after that he notices the number of ants has doubled. How many ants are in Mr. Elliott's kitchen in 10 days? (Show your work)

$5(2)^{10}$

$= 5120$

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9. 100 bacteria cells are observed under a microscope. The number of bacteria triples in size every 15 hours. How many cells of bacteria will there be in 75 hours?

$100(3)^{\frac{75}{15}}$

$= 24300.$

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10. Decide what type of relationship is given by each table below (linear, exponential or neither). **If the relationship is linear or exponential then give its equation.**

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a)

x	y
-1	5
0	1
1	-2
2	-1
3	0

neither

b)

x	y
0	2
1	6
2	18
3	54

exponential

$y = 2(3)^x$

c)

x	y
-1	7
0	5
1	3
2	1
3	-1

linear $y = -2x + 5$

11. The population of a city is growing exponentially. The population over a 4 year time period is given below. Find an equation that models the population of this city.

Year	Population
2000	95,000
2001	97,850
2002	100,786
2003	103,810

$\frac{97850}{95000} = 1.03$

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$P = 95000(1.03)^{n-2000}$

where P = population

n = year.