Unit 5 Test: Quadratic Equations MPM2D

Name: SOLUTIONS.

Marking Summary:

Total Marks: 55

Knowledge/Understanding: questions #1 - 3

Application: questions #4 - 6

Thinking/Inquiry and Problem Solving: #7, 8

Communication: all

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1. Solve each of the following quadratic equations. Round your final answer to two decimal places. **[8 marks]** (10)

a) $3x^2 + 23x = -8$

$3x^2+23x+8=0$

$x = \dfrac{-23 \pm \sqrt{23^2-4(3)(8)}}{2(3)}$

$x = \dfrac{-23 \pm \sqrt{433}}{6}$

$x \doteq -0.365$ or $x \doteq -7.3$

b) $5n = 2 - (2n+1)^2$

$5n = 2-(2n+1)(2n+1)$

$5n = 2-(4n^2+4n+1)$

$5n = 2-4n^2-4n-1$

$4n^2+4n+5n-2+1=0$

$4n^2+9n-1=0$

$n = \dfrac{-9 \pm \sqrt{9^2-4(4)(-1)}}{2(4)}$

$n = \dfrac{-9 \pm \sqrt{97}}{8}$

$n \doteq -2.36$ $n \doteq 0.11$

11/10

2. Use the discriminant to determine the number of x-intercepts for each parabola below. **[4 marks]**

a) $y = 2x^2 - x - 4$

$b^2-4ac = (-1)^2-4(2)(-4)$

$= 1+32$

$= 33$

2 x-intercepts

b) $y = -3x^2 + x - 5$

$b^2-4ac = 1^2-4(-3)(-5)$

$= -59$

< 0

∴ no x-intercepts.

3. A parabola is defined by the equation $y = -2x^2 - 4x + 30$. **[7 marks]**
Find the exact location of the following:
a) y-intercept b) zeroes c) vertex

a) (0, 30) ✓

b) $0 = -2x^2 - 4x + 30$

$0 = x^2 + 2x - 15$

$0 = (x+5)(x-3)$

$x+5 = 0$, or $x-3 = 0$

$x = -5$ or $x = 3$ ✓✓

(-5, 0) (3, 0)

c) vertex at

$x = \frac{-5+3}{2}$ ✓

$x = -1$

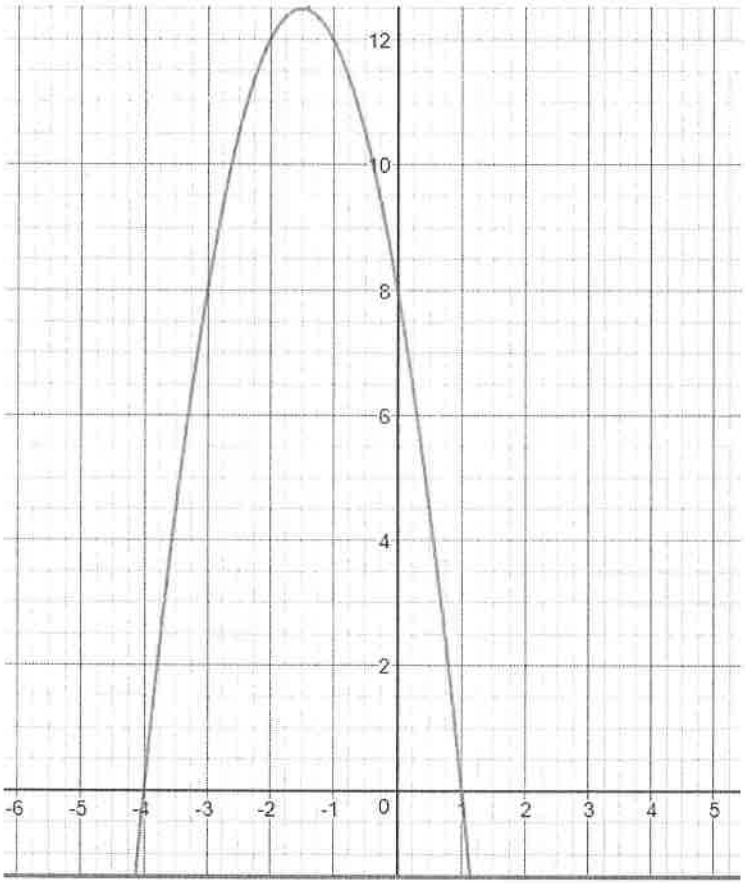
$y = -2(-1)^2 - 4(-1) + 30$

$y = -2 + 4 + 30$

$y = 32$

(-1, 32).

4. Find the equation of each parabola shown below. **[8 marks]**



$y = a(x-1)(x+4)$

sub in $(0, 8)$

$8 = a(-1)(4)$

$8 = -4a$

$a = -2$

$y = -2(x-1)(x+4)$

$y = a(x-2)^2 - 4$

sub in $(8, 8)$

$8 = a(8-2)^2 - 4$

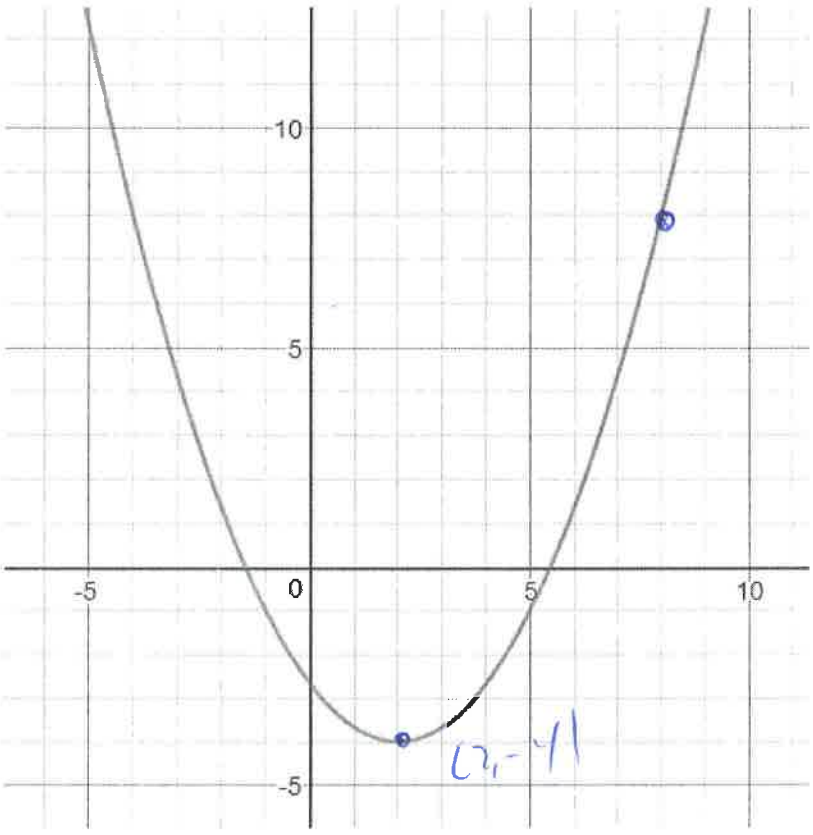
$8 = a(6)^2 - 4$

$8 = 36a - 4$

$12 = 36a$

$a = \frac{12}{36} = \frac{1}{3}$

$y = \frac{1}{3}(x-2)^2 - 4$



5. The path of a basketball after it is thrown is given by the equation $h = -0.25d^2 + 2d + 1.5$, where h is the height, in metres, and d is the horizontal distance, in metres.

a) How far has the ball travelled horizontally, to the nearest tenth of a metre, when it lands on the ground?

b) Find the maximum height of the basketball

[8 marks]

a) $0 = -.25d^2 + 2d + 1.5$

$d = \dfrac{-2 \pm \sqrt{2^2 - 4(-.25)(1.5)}}{2(-.25)}$

$d = \dfrac{-2 \pm \sqrt{5.5}}{-0.5}$

$d \doteq -0.69$ or $d \doteq 8.69$

$d > 0$ ∴ travelled 8.69m.

b) occurs at $d = \dfrac{-0.69 + 8.69}{2}$

$d = 4m$

$h = -0.25(4)^2 + 2(4) + 1.5$

$h = 5.5m$

∴ max height is 5.5m.

6. The population of deer in a national park can be modelled by the equation $P = 8n^2 - 112n + 570$, where P is the population of the deer and n is the number of years since the year 2010. (i.e. n = 0 corresponds to the year 2010).

a) Find the population of deer in the year 2015. **[6 marks]**
b) When will the deer population be 250?

a) $P = 8(5)^2 - 112(5) + 570$

$P = 210$

b) $250 = 8n^2 - 112n + 570$

$8n^2 - 112n + 320 = 0$

$n = \dfrac{112 \pm \sqrt{(-112)^2 - 4(8)(320)}}{2(8)}$

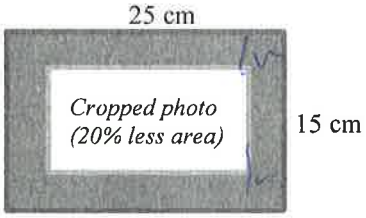
$n = \dfrac{112 \pm \sqrt{2304}}{16}$

$n = \dfrac{112 \pm 48}{16}$

$n = 10$ or $n = 4$

2010 and 2014.

7. A photograph measures 25 cm by 15 cm. Kerri plans to crop (cut) the photograph into a smaller photograph by cutting a strip of uniform width from each side of the photograph as shown in the diagram below. How wide should her cut be if she wishes to decrease the area of the photograph by 20%? **[6 marks]**



$A_{total} = 375 cm^2$ 80% of 375 $= 300 cm^2$.

$300 = (25 - 2w)(15 - 2w)$

$300 = 375 - 80w + 4w^2$

$4w^2 - 80w + 75 = 0$

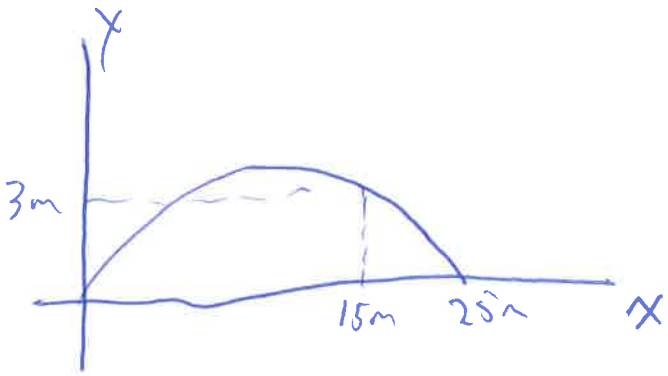
$w = \dfrac{80 \pm \sqrt{80^2 - 4(4)(75)}}{2(4)}$

$w = \dfrac{80 \pm \sqrt{5200}}{8}$

$w \doteq 19$ cm or $w \doteq 0.94$ cm

8. Jenny kicks a rugby ball through the posts (uprights) for a convert. The ball is kicked off the ground and travels a horizontal distance of 15m before just clearing the cross bar which has a height of 3m. The ball travels a total horizontal distance of 25 m.

   Find an equation the represents the path of the rugby ball in the air. Use your equation to determine the maximum height the ball reaches. (Give an exact answer). Include a sketch with your solution. **[6 marks]**



$y = a x(x-25)$

sub in $(15,3)$

$3 = a(15)(15-25)$

$3 = a(15)(-10)$

$3 = -150a$

$a = \frac{-3}{150}$

$a = -\frac{1}{50}$

$y = -\frac{1}{50}x(x-25)$

max height at $x = 12.5$m

$y = -\frac{1}{50}(12.5)(12.5-25)$

$y \doteq 3.125$ m.