Unit 2 Test

October 2019 MHF4U

Name: Solutions

Marking Scheme:

Knowledge/Understanding: Questions #2, 3,4, 6
Application Questions #1, 5
Thinking/Inquiry and Problem Solving: --
Communication: all

Question	
1	/14
2	/9
3	/8
4	/9
5	/9
6	/4
TOTAL:	**/53**

Answer the first question on this sheet of paper.

1. During the Apollo 14 mission to the moon, Alan Shepard hit a golf ball on the Moon. The function $h(t) = 18t - 0.8t^2$ models the height of the golf ball on the Moon, where *h(t)* is the height of the golf ball in metres, and *t* is time, in seconds.

   a) Find the average rate of change of the golf ball for the time interval. $5 \le t \le 10$ (include units). **[2 marks]**

   $$\frac{h(10)-h(5)}{5} = \frac{100-70}{5} = 6 \text{ m/s}$$

   $h(10) = 18(10) - 0.8(10)^2$
   $= 100$
   $h(5) = 70$

   b) Estimate the instananous rate of the change of the golf ball at 10 seconds, by calcuating the slopes of limiting secants. Do this by completing the table below. (you do not need to show full calculations) **[6 marks]**

Point 1	Point 2	Δt	$\Delta h(t)$	
(10, 100)	(10.1, 100.192)	0.1	0.192	1.92 m/s
(10, 100)	(10.01, 100.01992)	0.01	0.01992	1.992 m/s
(10, 100)	(9.9, 99.792)	-0.1	-0.208	2.08 m/s
(10, 100)	(9.99, 99.97992)	-0.01	-0.02008	2.008 m/s

Estimated rate of change at 10 seconds: 2 m/s

c) Use derivative "shortcuts" to find the exact rate of change at 10 seconds. (include units) **[3 marks]**

$h'(t) = 18 - 1.6t$

$h'(10) = 18 - 1.6(10)$

$h'(10) = 2$ m/s

d) Find the maximum height of the ball. **[3 marks]**

$h'(t) = 0 \rightarrow \quad 0 = 18 - 1.6t$

$1.6t = 18$

$t = 11.25$

$h(11.25) = 18(11.25) - 0.8(11.25)^2$

$= 101.25$

101.25 m

****Answer the remaining questions on foolscap paper and staple to test*****

2. Evaluate the following limits, if possible.

a) $\lim\limits_{x \to -1} \frac{x^2 + x}{x + 1}$

b) $\lim\limits_{x \to 0} \frac{2 - \sqrt{4 + x}}{x}$

c) $\lim\limits_{x \to 8} \frac{\sqrt[3]{x} - 2}{x - 8}$

3. a) Use the method of first principles to find the derivative of $f(x) = \frac{1}{2x - 1}$

b) Use your answer to part a) to evaluate $f'(2)$. Explain, in words, the meaning of $f'(2)$.

4. Differentiate the following (using "shortcuts"). Use the appropriate notation. Express final answers with positive exponents. Include radical signs when applicable.

a) $f(x) = x^3 - 4x^2 + 2x - 1$

b) $f(x) = x^4 - \frac{1}{x}$

c) $y = 5\sqrt{x^3} - \frac{1}{\sqrt[4]{x}}$

5. Let $f(x) = x^4 - 8x^2 + 2$.

a) Find the equation of the tangent line to the point (1, -5).

b) Find the location of all turning points of *f(x)*. Justify your solution by determining whether each turning point is a local minimum or local maximum.

2. a) $$\lim_{x\to -1} \frac{x^2+x}{x+1}$$

$$= \lim_{x\to -1} \frac{x(x+1)}{x+1}$$

$$= \lim_{x\to -1} x$$

$$= -1$$

b) $$\lim_{x\to 0} \frac{2-\sqrt{4+x}}{x}$$

$$= \lim_{x\to 0} \frac{2-\sqrt{4+x}}{x} \cdot \frac{2+\sqrt{4+x}}{2+\sqrt{4+x}}$$

$$= \lim_{x\to 0} \frac{4-(4+x)}{x(2+\sqrt{4+x})}$$

$$= \lim_{x\to 0} \frac{-x}{x(2+\sqrt{4+x})}$$

$$= \lim_{x\to 0} \frac{-1}{2+\sqrt{4+x}}$$

$$= \frac{-1}{4}.$$

c) $$\lim_{x\to 8} \frac{\sqrt[3]{x}-2}{x-8}$$

let $u=\sqrt[3]{x}$

$u^3=x$

as $x\to 8$, $u\to 2$

$$= \lim_{u\to 2} \frac{u-2}{u^3-8}$$

$$= \lim_{u\to 2} \frac{u-2}{(u-2)(u^2+2u+4)}$$

$$= \lim_{u\to 2} \frac{1}{u^2+2u+4}$$

$$= 1/12$$

3. a) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\frac{1}{2(x+h)-1} - \frac{1}{2x-1}}{h}$$

$$= \lim_{h \to 0} \frac{2x-1 - [2(x+h)-1]}{h\,(2(x+h)-1)(2x-1)}$$

$$= \lim_{h \to 0} \frac{\cancel{2x}-1-\cancel{2x}-2h+1}{h\,(2x+2h-1)(2x-1)}$$

$$= \lim_{h \to 0} \frac{-2h}{h\,(2x+2h-1)(2x-1)}$$

$$= \lim_{h \to 0} \frac{-2}{(2x+2h-1)(2x-1)}$$

$$= \frac{-2}{(2x-1)^2}$$

b) $f'(2) = \dfrac{-2}{(4-1)^2} = -2/9$

$-2/9$ is slope of tangent to graph at $x=2$.

or $-2/9$ is instaneous rate of change of graph at $x=2$.

4. a) $f'(x) = 3x^2 - 8x + 2$

b) $f'(x) = 4x^3 + x^{-2}$

$f'(x) = 4x^3 + \frac{1}{x^2}$

c) $y = 5x^{3/2} - x^{-1/4}$

$\frac{dy}{dx} = \frac{15}{2}x^{1/2} + \frac{1}{4}x^{-5/4}$

$\frac{dy}{dx} = \frac{15\sqrt{x}}{2} + \frac{1}{4\sqrt[4]{x^5}}$

5. $f'(x) = 4x^3 - 16x$ ✓

a) $f'(1) = 4 - 16$ ✓

$f'(1) = -12$

$-12 = \frac{y+5}{x-1}$ ✓

$-12x + 12 = y + 5$

$\boxed{y = -12x + 7}$

b) Let $f'(x) = 0$

$$0 = 4x^3 - 16x$$

$$0 = 4x(x^2 - 4)$$

$$0 = 4x(x-2)(x+2)$$

$x = 0$, $x = 2$ and $x = -2$ are critical points.

<u>check</u> $x = -2$

$f'(-3) = 4(-3)^3 - 16(-3) = -60 < 0$

$f'(-2) = 0$

$f'(-1) = 4(-1)^3 - 16(-1) = 12 > 0$

$\therefore$ $x = -2$ is a local min. $(-2, -14)$ min

<u>check</u> $x = 0$

$f'(-1) > 0$

$f'(0) = 0$

$(0, 2)$ max.

$f'(1) = 4 - 16 = -12 < 0$

$\therefore$ $x = 0$ is a local max.

check $x=2$

$f'(1) < 0$

$f'(2) = 0$

$f'(3) = 4(3)^3 - 16(3) = 60 > 0$

$\therefore$ $x=2$ a local min $(2, -14)$

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6. a) $f'(x) = 0$ at $x=-2$, $x=0$, $x=2$, $x=3$

b) $f'(x) > 0$ for: $x < -2$, $0 < x < 2$, $x > 3$.

(whenever graph is increasing).