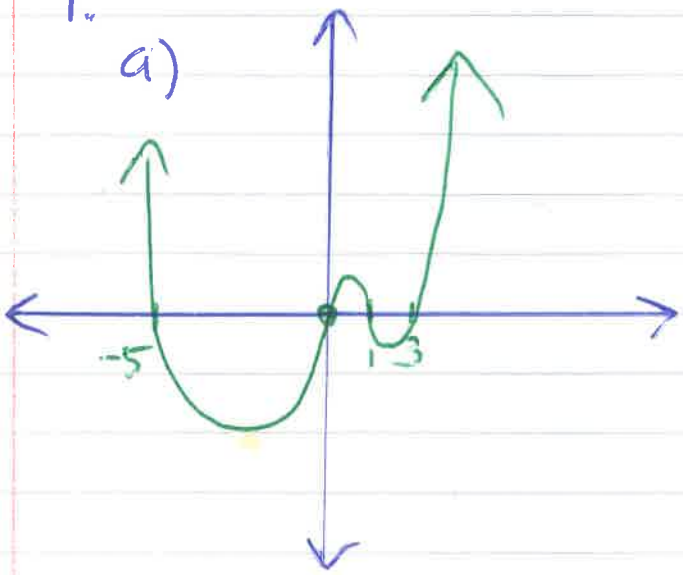
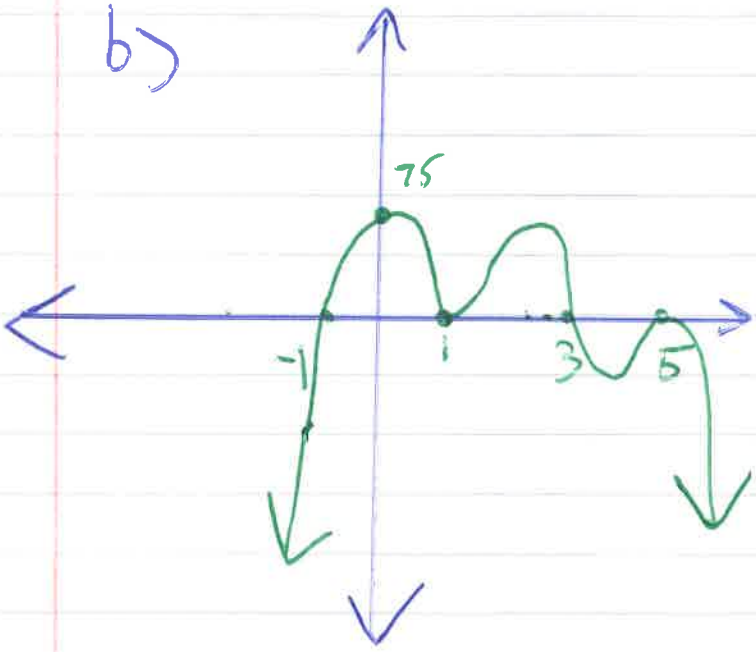
Unit 1 Test Solutions

1.

a)



b)



2.

a) $(f \circ g)(x) = \sqrt{x^2+1}$ ✓✓

b) $h[f(9)]$

$= h[\sqrt{9}]$

$= h(3)$

$= \frac{1}{3}$ ✓✓

c) $(h \circ g)(x) = \frac{1}{x^2+1}$

domain $\{x \in R\}$ (since $x^2+1 \neq 0$)

range $\{h \circ g \in R$ for $h \circ g \leq 1\}$. ✓✓

$x^2+1 \geq 1$

$\therefore \frac{1}{x^2+1} \leq 1$

and $h \circ g > 0$

since $1 > 0$

and $x^2+1 > 0$

3.

a) $x-3 \overline{)2x^3-4x^2+5x-2}$

$$\begin{array}{r} 2x^2+2x+11 \\ \hline 2x^3-6x^2 \\ \hline 2x^2+5x \\ 2x^2-6x \\ \hline 11x-2 \\ 11x-33 \\ \hline 31 \end{array}$$

$\therefore (2x^3-4x^2+5x-2) \div (x-3) = 2x^2+2x+11 + \frac{31}{x-3}$

b) $2x-1 \overline{)4x^4+4x^3-x^2-3x-7}$

$$\begin{array}{r} 2x^3+3x^2+x-1 \\ \hline 4x^4-2x^3 \\ \hline 6x^3-x^2 \\ 6x^3-3x^2 \\ \hline 2x^2-3x \\ 2x^2-x \\ \hline -2x-7 \\ -2x+1 \\ \hline -8 \end{array}$$

$\therefore (4x^4+4x^3-x^2-3x-7) \div (2x-1) = 2x^3+3x^2+x-1 + \frac{-8}{2x-1}$

4. $x^3 - 5 = 3x^2 - 7x$

$x^3 - 3x^2 + 7x - 5 = 0$

	1	-3	7	-5
x1		1	-2	5
	1	-2	5	0

$x = 1$ is a root.

$(x-1)(x^2 - 2x + 5) = 0$

$\boxed{x = 1}$ or $x^2 - 2x + 5 = 0$

$$x = \frac{2 \pm \sqrt{2^2 - 4(1)(5)}}{2}$$

$$x = \frac{2 \pm \sqrt{-16}}{2}$$

$$x = \frac{2 \pm 4i}{2}$$

$$\boxed{x = 1 \pm 2i}$$

5. $y = a(x+1)^2(x-2)(x-4)$

sub in $(3, -8)$

$-8 = a(3+1)^2(3-2)(3-4)$

$-8 = -16a$

$a = \frac{1}{2}$

$\therefore y = \frac{1}{2}(x+1)^2(x-2)(x-4)$

6. a) $8x^3+27$ $a = 2x$, $b = 3$

$= (2x+3)(4x^2-6x+9)$ ✓✓

$=$ ~~$(2x+3)(2x+3)(2x+3)$~~

b) $2x^4+3x^3-4x^2-6x$

$= x(2x^3+3x^2-4x-6)$ ✓✓

$= x(x^2(2x+3)-2(2x+3))$

$= x(x^2-2)(2x+3)$

$a = x^3$

c) $x^9-1 = (x^3-1)(x^6+x^3+1)$ ✓

$= (x-1)(x^2+x+1)(x^6+x^3+1)$

7. $x^3 - 5x^2 + 3x + 9 < 0$

$x = -1$ a "root"

$x^3 - 5x^2 + 3x + 9$

$= (x+1)(x^2 - 6x + 9)$

$= (x+1)(x-3)^2$

	1	−5	3	9
$x=-1$	↓	−1	6	−9
	1	−6	9	0

$x^3 - 5x^2 + 3x + 9 < 0$

$(x+1)(x-3)^2 < 0$

factors	regions $x < -1$	$-1 < x < 3$	$x > 3$
$(x-3)^2$	+	+	+
$(x+1)$	−	+	+
result	−	+	+

$\therefore \quad x < -1$

8. $x^6 + x^4 + x^2 + 1 < 0$

no solution. Even exponents with a +1 always ≥ 0.