

## Unit 3 Test: Logarithmic Functions

Name: \_\_\_\_\_

Page 1 – Complete this page without using a calculator. Once this page has been completed, hand it in to before pulling out your calculator. (You may use your calculator for the second page).

Question	
1	/15
2	/17
3	/5
4	/8
5	/9
6	/6
<b>TOTAL:</b>	<b>/60</b>

1. Evaluate the following expressions without using a calculator. Answer directly on page.

a)  $\log_3 \left( \frac{1}{81} \right)$

$$= -4$$

b)  $\log_2 \sqrt[4]{32}$

$$= \log_2 32^{\frac{1}{4}}$$

$$= \frac{1}{4} (5)$$

$$= \frac{5}{4}$$

c)  $\log_6 36 - \log_5 125$

$$= 2 - 3$$

$$= -1$$

d)  $\log_7 70 - \log_7 10$

$$= \log_7 \left( \frac{70}{10} \right)$$

$$= \log_7 7$$

$$= 1$$

e)  $9^{\log_9 150}$

$$= 150$$

f)  $\log 1$

$$= 0$$

g)  $\log_4 32 + \frac{1}{3} \log_4 8$

$$= \log_4 32 + \log_4 8^{\frac{1}{3}}$$

$$= \log_4 32 + \log_4 2$$

$$= \log_4 64$$

$$= 3$$

h)  $\log_4 \sqrt{32} - \log_4 \sqrt{8}$

$$= \log_4 \frac{\sqrt{32}}{\sqrt{8}}$$

$$= \log_4 \sqrt{4}$$

$$= \frac{1}{2} \log_4 4$$

$$= \frac{1}{2}$$

$$2. \quad a) \log x = 6$$

$$x = 10^6 \quad \checkmark \checkmark$$

$$x = 1\,000\,000$$

$$b) \log_x 16 = -\frac{4}{5}$$

$$x^{-\frac{4}{5}} = 16 \quad \checkmark$$

$$x = 16^{-\frac{5}{4}} \quad \checkmark \checkmark$$

$$x = \frac{1}{(\sqrt[4]{16})^5}$$

$$x = \frac{1}{32}$$

$$c) 3(2)^{5x+1} = 90$$

$$2^{5x+1} = 30$$

$$\log 2^{5x+1} = \log 30 \quad \checkmark \checkmark \checkmark$$

$$(5x+1) \log 2 = \log 30$$

$$x = \frac{\log 30}{5 \log 2} - \frac{1}{5}$$

$$x \approx 0.78$$

$$d) 3 \log x = \log 4 + \log 16$$

$$3 \log x = \log 64$$

$$\log x^3 = \log 64 \quad \checkmark \checkmark \checkmark$$

$$\text{so } x^3 = 64$$

$$x = \sqrt[3]{64}$$

$$x = 4.$$

$$e) \log_{12} (x-3) + \log_{12} (x+1) = 1$$

$$\log_{12} (x-3)(x+1) = 1$$

$$12^1 = (x-3)(x+1)$$

$$x^2 - 2x - 3 = 12$$

$$x^2 - 2x - 15 = 0$$

$\leftarrow$

$$(x-5)(x+3) = 0$$

$$\boxed{x=5} \quad x \neq -3 \quad (x > 3)$$

$\checkmark \checkmark \checkmark$

$$f) 2\log_3 x - \log_3 2x = \log_3 2$$

$$= \log_3 \left( \frac{x^2}{2x} \right) = \log_3 2$$

$$\frac{x^2}{2x} = 2$$

$$x^2 = 4x$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x=0, x=4$$

$$x \neq 0 \text{ so } x = 4.$$

$$3. P = 100(1.2)^{-t}$$

$$0.5 = 1.2^{-t}$$

$$\log 0.5 = \log 1.2^{-t}$$

$$\log 0.5 = -t \log 1.2$$

$$t = \frac{-\log 0.5}{\log 1.2}$$

$$t \approx 3.8 \text{ years.}$$

4.

$$C = -15 \log\left(\frac{T-20}{75}\right)$$

$$a) C = -15 \log\left(\frac{55-20}{75}\right)$$

$$C = -15 \log\left(\frac{35}{75}\right) \checkmark$$

$$C = -15 \log\left(\frac{7}{15}\right)$$

$$C = 5 \text{ minutes}$$

$$b) 0 = -15 \log\left(\frac{T-20}{75}\right)$$

$$0 = \log\left(\frac{T-20}{75}\right) \checkmark$$

$$10^0 = \frac{T-20}{75}$$

$$75 = T-20$$

$$T = 95$$

$$95^\circ \text{C}$$

$$c) \frac{C}{-15} = \log\left(\frac{T-20}{75}\right)$$

$$10^{\frac{-C}{15}} = \frac{T-20}{75}$$

$$75(10)^{\frac{-C}{15}} = T-20$$

$$T = 75(10)^{\frac{-C}{15}} + 20$$

$$\text{or } T = 75\left(\frac{1}{10^{\frac{C}{15}}}\right) + 20$$

5.  $f(x) = \log_5(x-2) - 1$

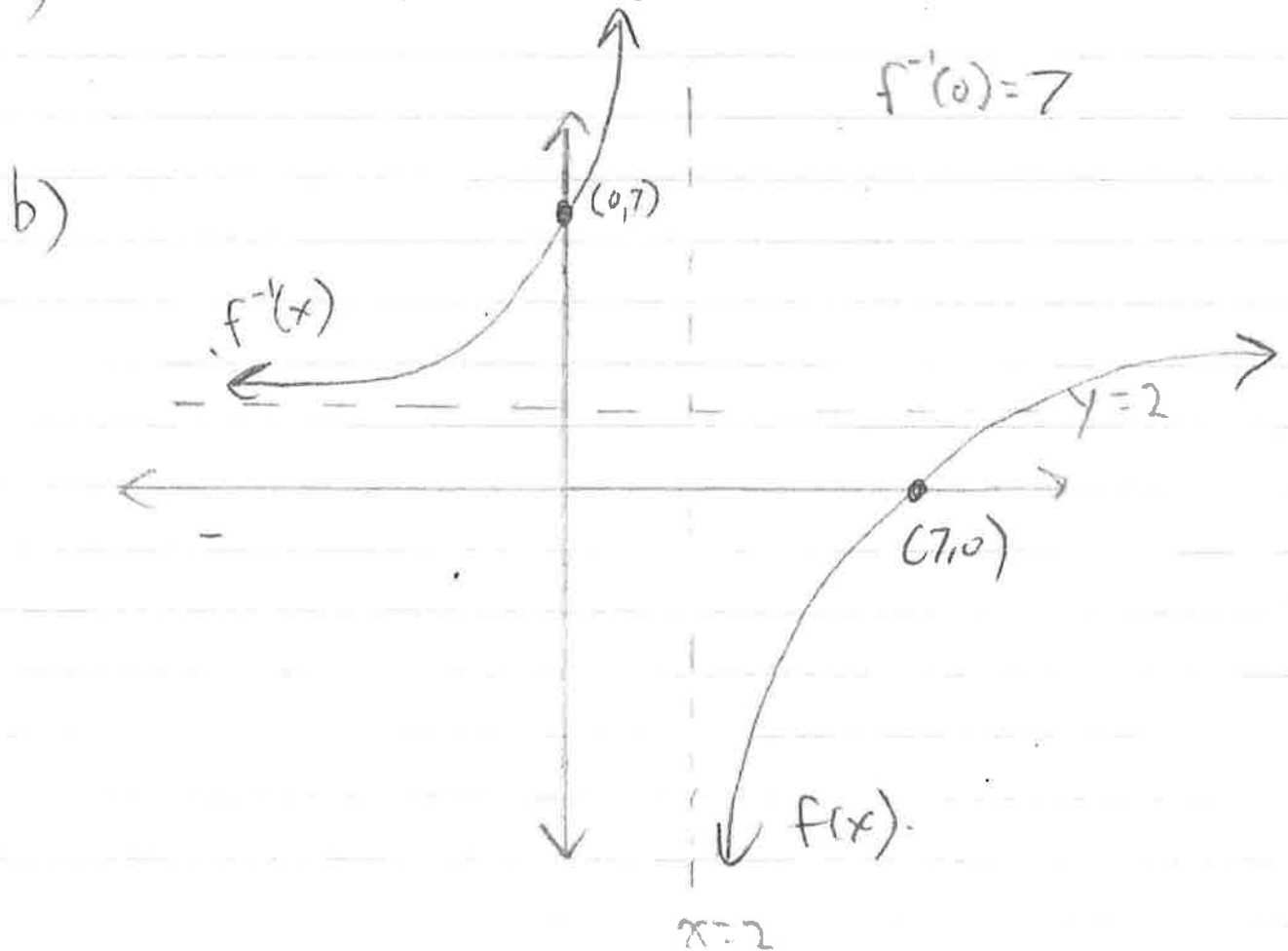
Inverse is  $x = \log_5(y+2) - 1$

$x+1 = \log_5(y+2)$  ✓✓

$5^{x+1} = y+2$

$y = 5^{x+1} + 2$

a) or  $f^{-1}(x) = 5^{x+1} + 2$



$$6. \quad 5^x = 3^{2x-1}$$

$$x \log 5 = (2x-1) \log 3$$

$$x \log 5 = 2x \log 3 - \log 3$$

$$x \log 5 - 2x \log 3 = -\log 3$$

$$x (\log 5 - 2 \log 3) = -\log 3$$

$$x = \frac{-\log 3}{\log 5 - 2 \log 3}$$

$$\text{or } x = \frac{\log 3}{2 \log 3 - \log 5}$$

$$\text{or } x = \frac{\log 3}{\log 9 - \log 5}$$

$$\text{or } x = \frac{\log 3}{\log \left(\frac{9}{5}\right)}$$

all are good!

$$x \approx 1.869$$