Unit #2 Test: Analytic Geometry MPM2D

Name: SOLUTIONS.

Marking Summary:

Total Marks: ______ 44

Knowledge/Understanding: questions #1-4

Application: questions #5-7

Thinking/Inquiry and Problem Solving: #8

Communication: all

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1. Find the exact distance between the points (1,2) and (-2, 7). **[4 marks]**

$$D = \sqrt{(-2-1)^2 + (7-2)^2}$$
$$= \sqrt{9+25}$$
$$= \sqrt{34}$$

2. Write the equation of a circle with a centre at (0,0) and a radius of 4. **[2 marks]**

$$x^2 + y^2 = 16$$

3. A line segment has endpoints C(2, -5) and D(-12, 7). Find the midpoint of line segment CD. **[3 marks]**

$$M_{CD}\left(\frac{2+-12}{2}, \frac{-5+7}{2}\right)$$
$$M_{CD}(-5, 1).$$

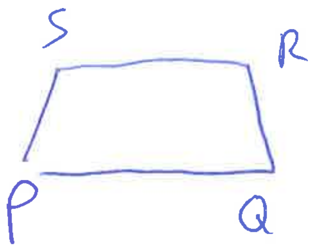
4. Mike draws a line segment from point A(1, 5) to point B(-8, 0). Marci draws a line segment from point P(-1,-3) to Q(10, -2). Which line segment is longest? **[4 marks]**

$$AB = \sqrt{(-8-1)^2 + (0-5)^2}$$
$$= \sqrt{81+25}$$
$$= \sqrt{106}$$

$$PQ = \sqrt{(10-(-1))^2 + (-2-(-3))^2}$$
$$= \sqrt{11^2 + 1^2}$$
$$= \sqrt{122}$$

$\therefore$ Marci's is longer.

5. James draws 4 points on a graph. Point P is at (-1, -3), point Q is at (6, -8), point R is at (19, -4) and point S is at (5, 6). He connects the points from P to Q to R to S to make quadrilateral PQRS. Show that PQRS is a trapezoid. **[4 marks]**



$$m_{PQ} = \frac{-8-(-3)}{6-(-1)}$$

$$m_{PQ} = \frac{-5}{7}$$

$$m_{RS} = \frac{6-(-4)}{5-19}$$

$$= \frac{10}{-14}$$

$$m_{RS} = -5/7$$

$\therefore$ PQRS is a trapezoid with PQ // RS

note (not needed)

$$m_{PS} = \frac{6-(-3)}{5-(-1)}$$

$$= \frac{9}{6}$$

$$= 3/2$$

$$m_{QR} = \frac{-4-(-8)}{19-6}$$

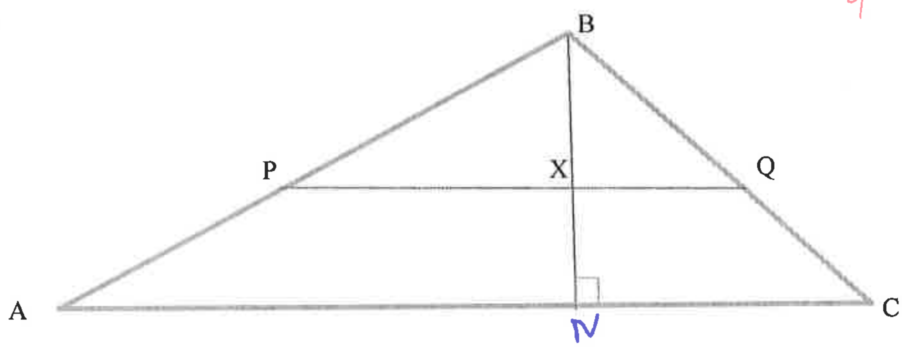
$$= \frac{4}{13}.$$

6. Triangle ABC has vertices A(-6,-4), B(0,4) and C(8,0). A mid-segment and alitutde have been drawn inside triangle ABC. **[12 marks]**

   a) A mid-segment connects two midpoints of a triangle. Find the equation for the mid-segment PQ, shown in the diagram below. /4

   b) An altitude is drawn perpendicular to side AC to vertex B as shown. Find the equation of this altitude. /4

   c) The altitude from part b) and the mid-segment from part a) intersect at point X. Find the coordinates of point X. /4



a) $P\left(\frac{-6+0}{2}, \frac{-4+4}{2}\right)$

$P(-3, 0)$

$Q\left(\frac{0+8}{2}, \frac{4+0}{2}\right)$

$Q(4, 2)$.

$m_{PQ} = \frac{2-0}{4-(-3)}$

$m_{PQ} = \frac{2}{7}$

Eqn for PQ: $\frac{2}{7} = \frac{y-0}{x+3}$

$2x + 6 = 7y$

$\boxed{2x - 7y + 6 = 0}$

b) $m_{AC} = \frac{0-(-4)}{8-(-6)}$

$= \frac{4}{14}$

$= \frac{2}{7}$. (of course!).

$\therefore m_{BN} = -7/2$ (altitude slope)

Eqn for altitude BN: $\frac{-7}{2} = \frac{y-4}{x-0}$

$-7x = 2y - 8$

$\boxed{7x + 2y - 8 = 0}$.

or $y = -\frac{7}{2}x + 1$

c) Find x

$$① \; 2x - 7y + 6 = 0 \xrightarrow{\times 7} 14x - 49y + 42 = 0$$
$$② \; 7x + 2y - 8 = 0 \xrightarrow{\times 2} 14x + 4y - 16 = 0$$

$$-53y + 58 = 0$$

$$y = \frac{58}{53}$$

$$2x - 7\left(\frac{58}{53}\right) + 6 = 0$$

$$2x - \frac{406}{53} + 6 = 0$$

$$2x - \frac{406}{53} + \frac{318}{53} = 0$$

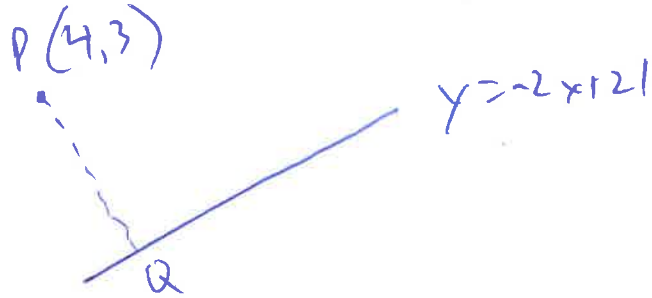
$$2x - \frac{88}{53} = 0$$

$$2x = \frac{88}{53}$$

$$x = \frac{44}{53}$$

∴ X is at $\left(\frac{44}{53}, \frac{58}{53}\right)$

7. Find the shortest distance from the line $y = -2x + 21$ to the point (4, 3). **[7 marks]**



① Find eqn for PQ. $m_{PQ} = \frac{1}{2}$.

$$\frac{1}{2} = \frac{y-3}{x-4}$$

$$x - 4 = 2y - 6$$

$$x - 2y = -2$$

② Find Q

① $y = -2x + 21$

② $x - 2y = -2$

sub ① into ②

$$x - 2(-2x + 21) = -2$$

$$x + 4x - 42 = -2$$

$$5x = 40$$

$$x = 8$$

$$y = -2(8) + 21$$

$$y = 5$$

Q (8, 5)

③ $PQ = \sqrt{(8-4)^2 + (5-3)^2}$

$PQ = \sqrt{16 + 4}$

$PQ = \sqrt{20}$

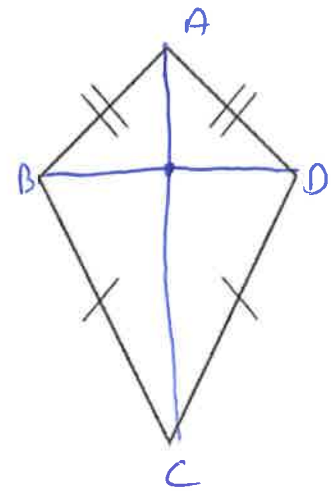
$\therefore$ the distance is $\sqrt{20}$.

8. A kite is a quadrilateral with adjacent sides having equal length (as shown below). The vertices of a kite are A(-2,3), B(1,-1), C(4,0) and D(3,3).

[8 marks]

a) Verify that the diagonals are perpendicular.

b) Show that diagonal AC bisects ("cuts in half") diagonal BD.



a) $m_{AC} = \frac{0-3}{4-(-2)}$

$= \frac{-3}{6}$

$= -\frac{1}{2}$

$m_{BD} = \frac{3-(-1)}{3-1}$

$= \frac{4}{2}$

$= 2$

$\therefore AC \perp BD.$

b) $M_{BD}\left(\frac{1+3}{2}, \frac{-1+3}{2}\right)$

$M_{BD}(2, 1).$

Eqn for AC:

$-\frac{1}{2} = \frac{y-0}{x-4}$

$2y = -x + 4$

$y = -\frac{1}{2}x + 2$

sub in (2,1)

$LS = 1 \qquad RS = -\frac{1}{2}(2) + 2$

$= -1 + 2$

$= 1$

$\therefore$ (2,1) is P.O.I of diagonals and midpoint of BD.

$\therefore$ BD is bisected by AC.