

Unit #1 Test: Linear Systems

MPM2D

Name: solutions

Marking Summary:

Total Marks: _____

50

Knowledge/Understanding: questions #1 - 5

Application: questions #6-7

Thinking/Inquiry and Problem Solving: #8 - 9

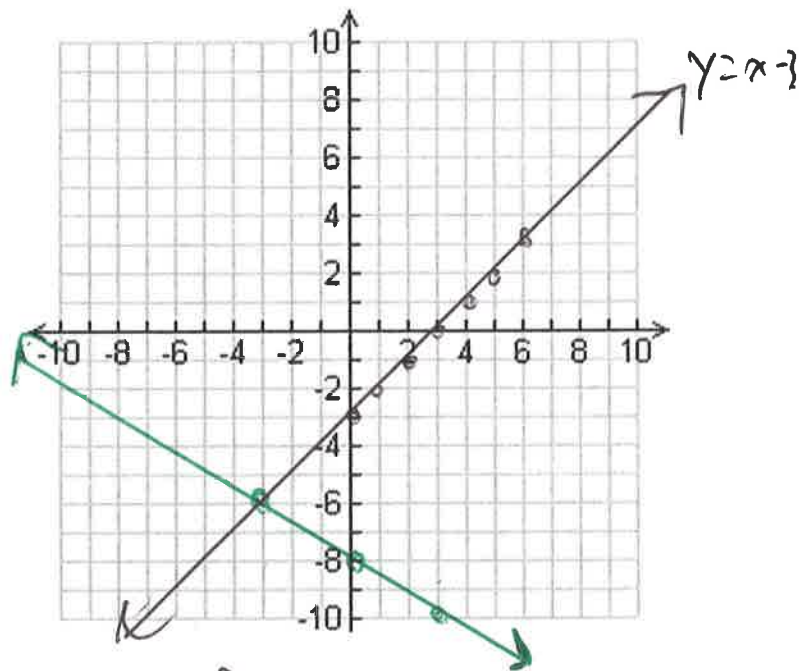
Communication: all

1. Solve each linear system below by graphing. [8 marks]

a) $y = x - 3$

$$y = \frac{-2}{3}x - 8$$

Soln is $(-3, -6)$



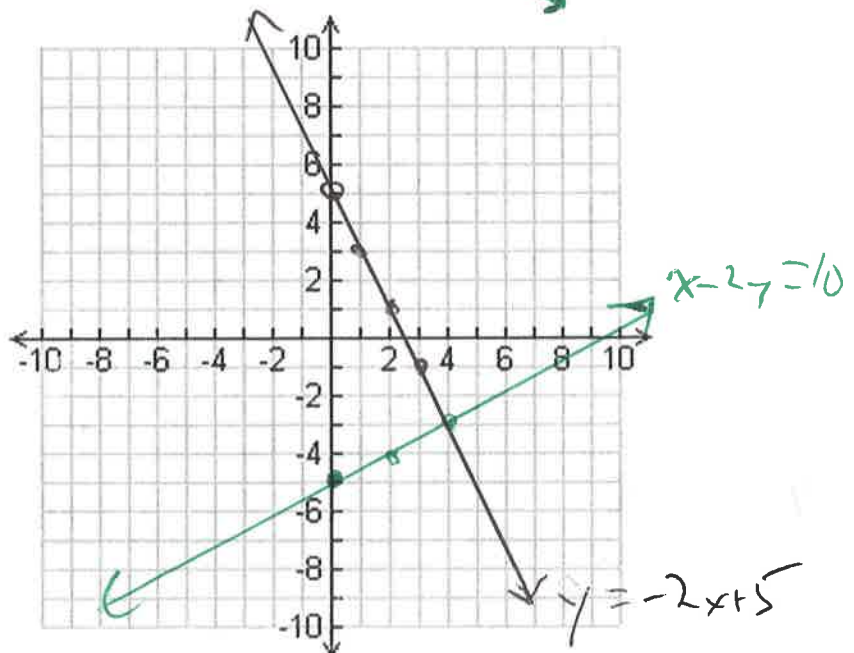
b) $2x + y = 5 \rightarrow y = -2x + 5$

$$x - 2y = 10$$

$$-2y = -x + 10$$

$$y = \frac{1}{2}x - 5$$

Soln is $(4, -3)$



2. Solve the following linear system using substitution. [4 marks]

$$\textcircled{1} 2x - y = 3$$

$$\textcircled{2} 4x + y = 9 \rightarrow y = 9 - 4x$$

$$y = 9 - 4(2)$$

$$y = 1$$

s. b. $\textcircled{2}$ into $\textcircled{1}$

$$2x - (9 - 4x) = 4$$

$$2x - 9 + 4x = 4$$

$$6x - 9 = 4$$

$$6x = 12$$

$$x = 2$$

$$(2, 1)$$

3. Solve the following linear system using elimination. [4 marks]

$$4x + 3y = 13 \xrightarrow{\times 5} 20x + 15y = 65$$

$$5x - 4y = -7 \xrightarrow{\times 4} -20x - 16y = -28$$

$$\hline 31y = 93$$

$$y = 3$$

$$4x + 3(3) = 13$$

$$4x + 9 = 13$$

$$4x = 4$$

$$x = 1$$

4. Use either substitution or elimination to solve the following. [4 marks]

$$3a - 2b + 4 = 0 \xrightarrow{\times 2} 6a - 4b + 8 = 0$$

$$2a - 5b - 1 = 0 \xrightarrow{\times 3} 6a - 15b - 3 = 0$$

$$\hline 11b + 11 = 0$$

$$\boxed{b = -1}$$

$$3a - 2(-1) + 4 = 0$$

$$3a + 6 = 0$$

$$\boxed{a = -2}$$

5. Use either substitution or elimination to solve each of the following. [8 marks]

$$\begin{array}{l} \text{a) } 4 + 2y = -4x \rightarrow 4x + 2y = -4 \rightarrow 4x + 2y = -8 \\ 2x + y = 9 \rightarrow 2x + y = 9 \xrightarrow{\times 2} 4x + 2y = 18 \end{array}$$

0 ≠ 26

∅ no solution.

$$\begin{array}{l} \text{b) } 6x - y = -1 \rightarrow 6x - y = -1 \xrightarrow{\times 3} 18x - 3y = -3 \\ 3y - 8x = 2 \rightarrow -8x + 3y = 2 \xrightarrow{\times 3} -24x + 9y = 6 \\ \hline 10x = -1 \\ \boxed{x = -\frac{1}{10}} \end{array}$$

$$\begin{array}{l} 6\left(-\frac{1}{10}\right) - y = -1 \\ -\frac{3}{5} - y = -1 \\ -y = -1 + \frac{3}{5} \\ -y = -\frac{2}{5} \\ \boxed{y = \frac{2}{5}} \end{array}$$

6. Melissa takes her family to a burger stand. She orders 3 hot dogs and 2 hamburgers. The total cost is \$15.95. The next week she returns to the same burger stand. This time she orders 4 hot dogs and 3 hamburgers and the total cost is \$22.55.

What is the cost of a hot dog? What is the cost of a hamburger? [6 marks]

Let $b = \text{cost of burger } (\$)$, $d = \text{cost of hot dog } (\$)$

$$\begin{array}{l} 3d + 2b = 15.95 \xrightarrow{\times 4} 12d + 8b = 63.80 \\ 4d + 3b = 22.55 \xrightarrow{\times 3} 12d + 9b = 67.65 \end{array}$$

$$-b = -3.85$$

$$b = 3.85$$

$$4d + 3(7.85) = 22.55$$

$$4d + 11.55 = 22.55$$

$$4d = 11$$

$$d = 2.75$$

hot dog is \$2.75
burger is \$3.85

7. One type of granola has 30% nuts, by mass. A second type of granola has 15% nuts. What mass of each type must be mixed to make 600g of granola that will have 21% nuts? [6 marks]

Let $x = 30\%$ nuts (in g) $y = 15\%$ nuts (in g)

$$x + y = 600 \quad \rightarrow \quad x + y = 600 \rightarrow x = 600 - y \quad \textcircled{1}$$

$$-3x + .15y = 600(0.21) \rightarrow -3x + .15y = 126 \quad \textcircled{2}$$

sub $\textcircled{1}$ into $\textcircled{2}$ $.3(600 - y) + .15y = 126$

$$180 - 0.3y + 0.15y = 126$$

$$-0.15y = -54$$

$$y = 360$$

$$x = 600 - 360$$

$$x = 240$$

so 240g of 30% mix and
360g of 15% mix.

8. Kerri works for an automotive mechanic. She has two "jobs": installing new tires and performing an oil change. Installing tires takes her 45 minutes. An oil change takes her 20 minutes. One particular week she logs 64 jobs taking a total of 30.5 hours.

How many times did she install a new set of tires? [6 marks]

Let $t =$ number of tire installations
 $c =$ number of oil changes.

$$45 \text{ mins} = 0.75 \text{ hours}$$

$$20 \text{ mins} = \frac{1}{3} \text{ h}$$

$$\textcircled{1} \quad t + c = 64 \rightarrow t = 64 - c$$

~~37c + 20c = 1830~~

$$\textcircled{2} \quad 45t + 20c = 1830$$

$$30.5 \text{ hours} = 1830 \text{ mins}$$

sub $t = 64 - c$ into $\textcircled{2}$

$$45(64 - c) + 20c = 1830$$

$$2880 - 45c + 20c = 1830$$

$$-25c = -1050$$

$$c = 42$$

$$t = 64 - 42$$

$$t = 22$$

so 22 tire changes and 42 oil changes.

9. Erik goes kayaking down a river. He starts by kayaking down the river (with the current) and travels 25 km in 2.5 hours. Erik then paddles up the river (against the current). After 3 hours he has only travelled 15 km up stream before getting tired. Assuming he paddled at the same water speed in both directions, how fast is the current? How fast could Erik travel with no current? [6 marks]

Let k = speed of kayak (in still water) km/h
 c = current speed (km/h)

$$\textcircled{1} 2.5(k+c) = 25$$

$$\textcircled{2} 3(k-c) = 15 \quad \xrightarrow{\div 3} \quad k-c = 5 \rightarrow k = 5+c$$

Sub. $\textcircled{2}$ into $\textcircled{1}$

$$2.5(5+c+c) = 25$$

$$2.5(5+2c) = 25$$

$$12.5 + 5c = 25$$

$$5c = 12.5$$

$$c = 2.5$$

$$k = 5 + 2.5$$

$$k = 7.5$$

∴ Current is 2.5 km/h, kayak can go 7.5 km/h w/ no current.