Unit 5 - Test Solutions

① $f(x) = \dfrac{x^2+2x+3}{x-1}$

$f(0) = -3$ $\qquad$ $0 = f(x) \to 0 = x^2+2x+3$

not possible $\quad 2^2-4(3) < 0.$

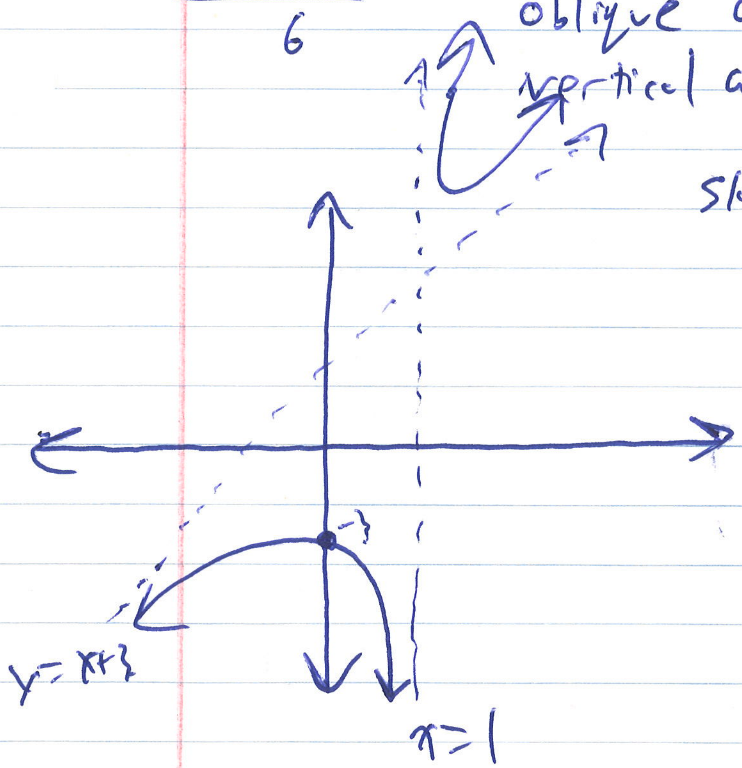
$$\begin{array}{r|l} & x+3 \\ \hline x-1 & x^2+2x+3 \\ & x^2-x \\ \hline & 3x+3 \\ & 3x-3 \\ \hline & 6 \end{array}$$

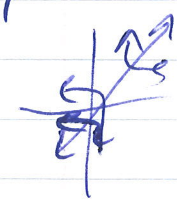
$f(x) = x+3+\dfrac{6}{x-1}$

oblique asymptote at $y = x+3$

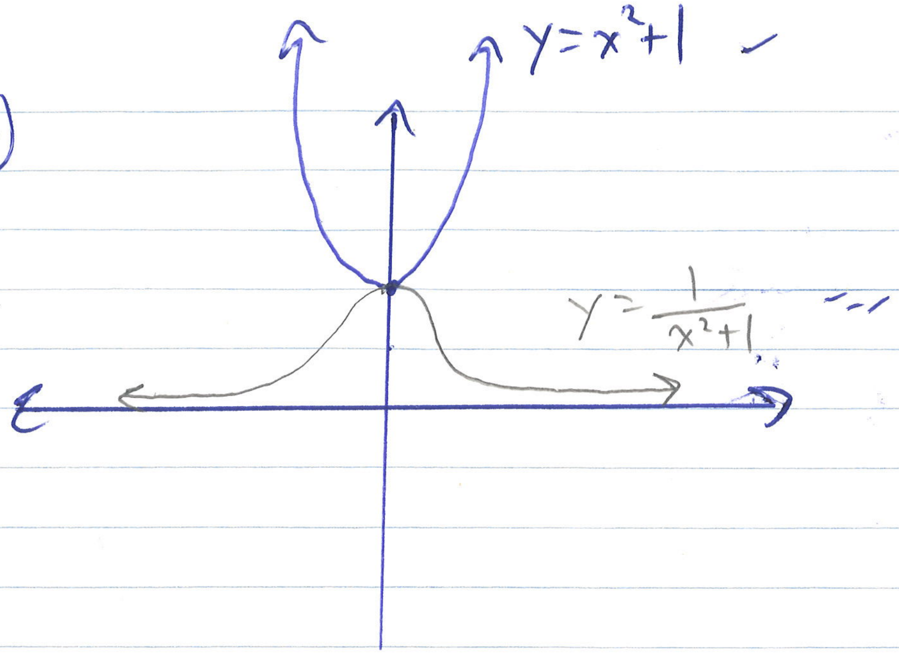
vertical asymptote at $x = 1$

sketch





(2)



② a) $\dfrac{2x}{(x+3)(x+1)} = \dfrac{A}{x+3} + \dfrac{B}{x+1}$

$2x = A(x+1) + B(x+3)$

let $x=-1$

$-2 = A(0) + 2B$

$B = -1$

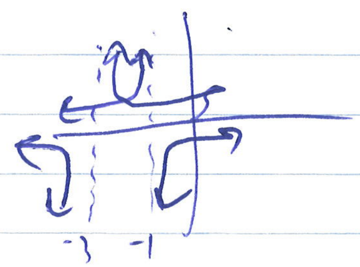
let $x=-3$

$2(-3) = A(-2) + B(0)$

$-6 = -2A$

$A = 3.$

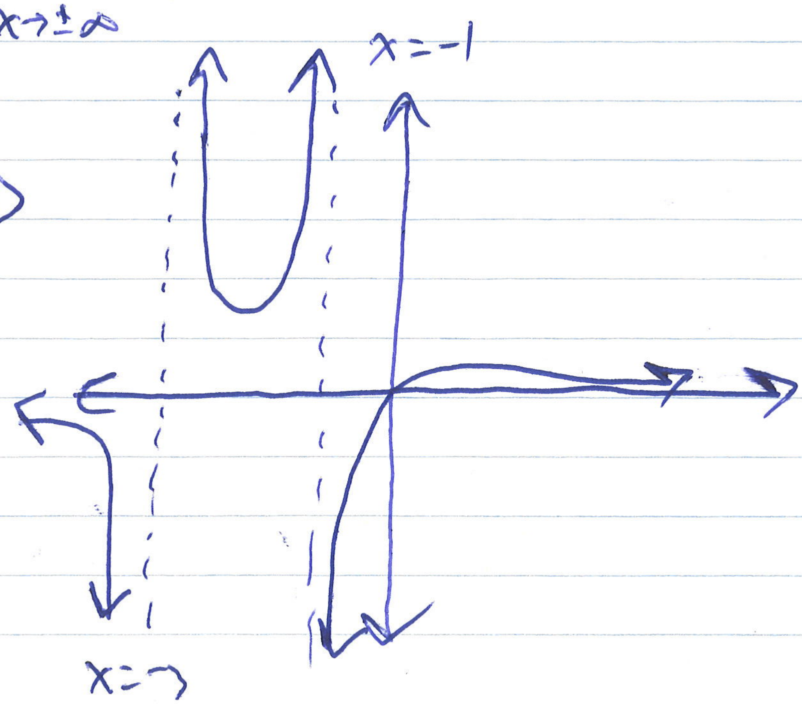
b) $f(x) = \dfrac{3}{x+3} - \dfrac{1}{x+1}$



V.A. at $x=-3$, $x=-1$

H.A. at $y=0$ since $\lim\limits_{x\to\pm\infty} f(x) = 0$

Only intercept is $(0,0)$



$\lim\limits_{x\to+\infty} \dfrac{2x}{(x+3)(x+1)} = 0^+$

$\lim\limits_{x\to-\infty} \dfrac{2x}{(x+3)(x+1)} = 0^-$

4.

i) $f(x) = \frac{x-5}{x^2-7x+10}$

c) hole at $(5, \frac{1}{3})$

$f(x) = \frac{\cancel{x-5}}{(\cancel{x-5})(x-2)}$

b) V.A. at $x=2$

H.A. at $y=0$

$= \frac{1}{x-2} \quad x \neq 5$

a) $\lim_{x \to +\infty} f(x) = 0^+$

$\lim_{x \to -\infty} f(x) = 0^-$

ii) $y = \frac{5x^2+1}{x^2-4}$

b) V.A. at $x=2$ and $x=-2$

H.A. at $y=5$

c) no holes

a) $\lim_{x \to +\infty} \frac{5x^2+1}{x^2-4} = 5^+$

$\lim_{x \to -\infty} \frac{5x^2+1}{x^2-4} = 5^+$

iii) $g(x) = \frac{2x^2-3x+1}{x+4}$

$$\begin{array}{r}
2x-11 \\
x+4 \overline{)\,2x^2-3x+1} \\
2x^2+8x \\
\hline
-11x+1 \\
-11x-44 \\
\hline
+45
\end{array}$$

$$g(x) = 2x - 11 + \frac{45}{x+4}$$

b) V.A. at $x = -4$

oblique/slant asymptote at $y = 2x - 11$

c) no holes

a) $\lim_{x \to +\infty} g(x) = +\infty$

$\lim_{x \to -\infty} g(x) = -\infty$

5. $\frac{3}{x} - \frac{6}{x+3} = \frac{1}{2x}$

$2x(x+3)\frac{3}{x} - 2x(x+3)\frac{6}{x+3} = \frac{2x(x+3)}{2x}$ $x \neq 0, -3$

$6x + 18 - 12x = x + 3$

$-7x = -15$

$x = 15/7$

6.

Let t = time stuart takes to deliver flyers (minutes)

$$\frac{1}{t} + \frac{1}{t-13} = \frac{1}{42}$$ $42t(t-13)$

$$42(t-13) + 42t = t(t-13)$$

$$42t - 546 + 42t = t^2 - 13t$$

$$t^2 - 97t + 546 = 0$$

$$(t-91)(t-6) = 0$$ (or use quad. formula)

$t = 91$ or $t = 6$

Note $t > 13$. $\therefore t = 91$

Stuart takes 91 minutes to deliver flyers.

7.

a) $\dfrac{(x+3)(x-2)}{x-4} \geq 0.$ $-3, 2, 4.$

$x \neq 4.$

factors	regions. $x<-3$	$-3<x<2$	$2<x<4$	$x>4$
$x+3$	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$	$+$	$+$
$x-4$	$-$	$-$	$-$	$+$
result	$-$	$+$	$-$	$+$

∴ solution is $-3 \leq x \leq 2$ and $x>4$

b) $\dfrac{-x}{x-1} + \dfrac{3}{x+7} \geq 0$

$\dfrac{-x(x+7)+3(x-1)}{(x-1)(x+7)} \geq 0$

$\dfrac{-x^2-7x+3x-3}{(x-1)(x+7)} \geq 0$

$\dfrac{-x^2-4x-3}{(x-1)(x+7)} \geq 0$

$-7, -3, -1, 1$

$$\frac{-(x+3)(x+1)}{(x-1)(x+7)} \ge 0 \qquad x \neq 1, -7$$

factors	regions $x<-7$	$-7<x<-3$	$-3<x<-1$	$-1<x<1$	$x>1$
-1	−	−	−	−	−
$x+3$	−	−	+	+	+
$x+1$	−	−	~~+~~ −	+	+
$x-1$	−	−	−	−	+
$x+7$	−	+	+	+	+
result	−	+	−	+	−

solution: $-7 < x \le 3$ and $-1 \le x < 1$