MCV4U

Introduction to Vectors

Lesson 1

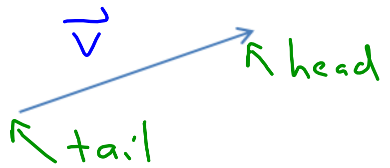
A **scalar** is a quantity having ***magnitude*** only.

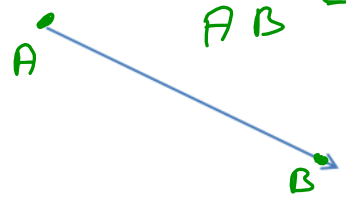
Examples: any real number
speed (km/h)
mass (kg)
length (cm)

A **vector** refers to a quantity that has both ***magnitude*** and ***direction***.

Examples: velocity eg. 45 km/h North
forces

We will begin this course by representing vectors ***geometrically***. They are represented as a line segment with direction (a ***directed line segment)***.





$\overrightarrow{AB}$ ← point to point vector.

How do we express:

The magnitude of a vector?

equal to length of the line segment.

$|\vec{v}|$ $|\overrightarrow{AB}|$

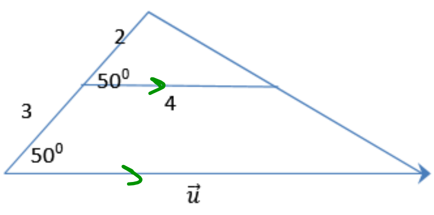
The direction of a vector?

direction it makes with respect to another vector or coordinate system. angle

direction between 2 vectors is angle made when vectors are placed tail-to-tail

Examples: Find $|\vec{u}|$ in each of the following.

a)

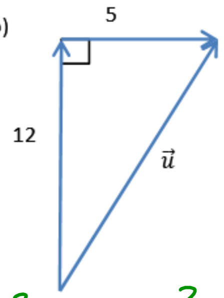


$$\frac{|\vec{u}|}{4} = \frac{5}{2}$$

$$2|\vec{u}| = 20$$

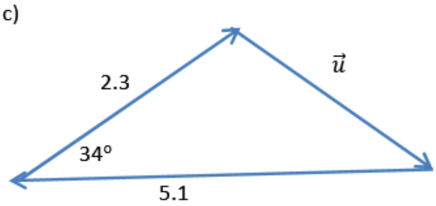
$$|\vec{u}| = 10$$

b)



$$|\vec{u}|^2 = 12^2 + 5^2$$
$$|\vec{u}| = 13$$

c)



$$|\vec{u}|^2 = 5.1^2 + 2.3^2 - 2(5.1)(2.3)\cos 34^\circ$$
$$|\vec{u}| \doteq 3.4$$

Equality of Vectors

Two vectors, $\vec{u}$ and $\vec{v}$ are equal if and only if:

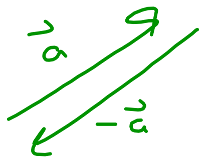
1) $|\vec{u}| = |\vec{v}|$

and

2) $\vec{u}$ and $\vec{v}$ have the same direction.

We can then say that: $\vec{u} = \vec{v}$

The Negative of a Vector



The negative of a vector is a vector with the **same magnitude** but **opposite direction**.

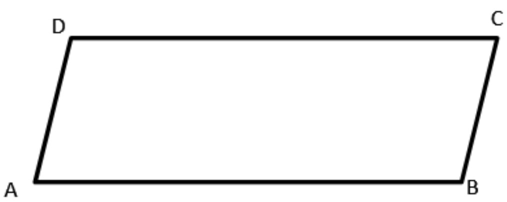
We can say that the negative of $\overrightarrow{AB}$

$$-\overrightarrow{AB} = \overrightarrow{BA}$$

The Zero Vector

The zero vector has a magnitude of zero. Its direction is undefined. $\vec{0}$

Example: In parallelogram ABCD, find a vector equal to:



a) $\overrightarrow{AB}$ $= \overrightarrow{DC}$

b) $\overrightarrow{DA}$ $= \overrightarrow{CB}$ $= -\overrightarrow{BC}$

c) $-\overrightarrow{CD}$ $= \overrightarrow{DC}$ $= \overrightarrow{AB}$

Find vectors equal to the negative of:

d) $\overrightarrow{AB}$

$\overrightarrow{BA}$, $\overrightarrow{CD}$

e) $\overrightarrow{CB}$

$\overrightarrow{AD}$

Scalar Multiplication

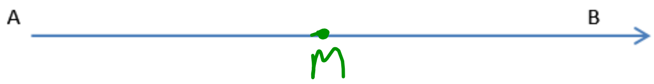
Recall that a scalar quantity can be any real number. $k \in \mathbb{R}$

A vector $\vec{v}$ can be multiplied by a scalar, k, to produce a new vector $k\vec{v}$ such that:

1) $|k\vec{v}| = |k|\,|\vec{v}|$

2) direction of $k\vec{v}$ is same as $\vec{v}$ if $k>0$ and opposite if $k<0$.

Example: Given M is the midpoint of $\overrightarrow{AB}$, express each vector below as a scalar multiple of another. (Label the diagram first)



a) $\overrightarrow{AB} = 2\overrightarrow{AM}$
$= 2\overrightarrow{MB}$

b) $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BA}$
$= -\frac{1}{2}\overrightarrow{AB}$

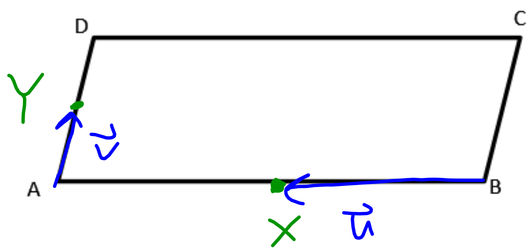
Example ABCD is a parallelogram with X and Y as midpoints of AB and AD, respectively. If $\vec{u} = \overrightarrow{BX}$ and $\vec{v} = \overrightarrow{AY}$ express the following in terms of $\vec{u}$ and $\vec{v}$.

a) $\overrightarrow{AD} = 2\vec{v}$

b) $\overrightarrow{XA} = \vec{u}$

c) $\overrightarrow{CD} = 2\vec{u}$

d) $\overrightarrow{CB} = -2\vec{v}$



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