Section 4.2 pg 133.

1. a) $\vec{u}+\vec{v}=\overrightarrow{DB}$ (//gram law)

$\vec{u}-\vec{v} = \vec{u}+-\vec{v}$

$= \vec{DC}+\vec{AD}$

$= \vec{AD}+\vec{DC}$

$= \vec{AC}$ (triangle law).

b) $\vec{u}+\vec{v}=\vec{v}+\vec{u}=\overrightarrow{CA}$ (triangle law)

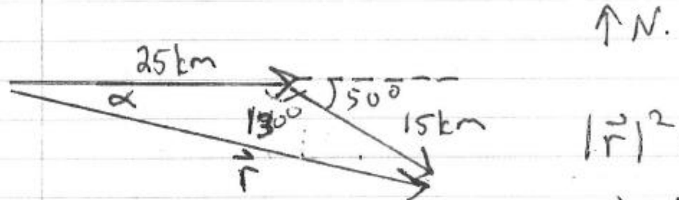
$\vec{u}-\vec{v}=\vec{u}+(-\vec{v})=\overrightarrow{BD}$ (//gram law)

c) $\vec{u}-\vec{v}=\vec{u}+(-\vec{v})$

$=\vec{AB}$ (triangle law).

don't worry about $\vec{u}+\vec{v}$. (poor question).

5. S50E → "50° South of East"



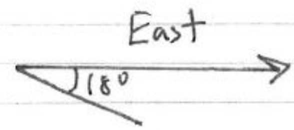
$$|\vec{r}|^2 = 25^2 + 15^2 - 2(25)(15)\cos 130^\circ$$

$$|\vec{r}| \doteq 36.5\text{ km}$$

$$\frac{\sin\alpha}{15} = \frac{\sin 130^\circ}{36.5}$$

$$\sin\alpha = \frac{15\sin 130^\circ}{36.5}$$

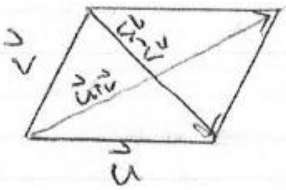
$$\alpha \doteq 18^\circ$$



∴ resultant vector has magnitude 36.5 km and direction S18°E ("18° South of East")

Wrong answer in text?

7. a) $|\vec{u}+\vec{v}| = |\vec{u}-\vec{v}|$ diagonals are equal.
$\parallel$gram is a rectangle.

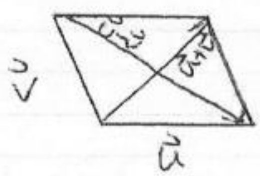


$\therefore \vec{u} \perp \vec{v}$.

b) $|\vec{u}+\vec{v}| > |\vec{u}-\vec{v}|$ as shown above.

$\therefore 0 < \theta < 90^\circ$

c) $|\vec{u}+\vec{v}| < |\vec{u}-\vec{v}|$ shown below.



$\therefore 90^\circ < \theta < 180^\circ$.

<u>OR</u> $|\vec{u}+\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 \boxed{+ 2|\vec{u}||\vec{v}|\cos\theta}$

and $|\vec{u}-\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 \boxed{-2|\vec{u}||\vec{v}|\cos\theta}$

$2|\vec{u}||\vec{v}|\cos\theta = -2|\vec{u}||\vec{v}|\cos\theta$ if $\theta = 90^\circ$

$2|\vec{u}||\vec{v}|\cos\theta > -2|\vec{u}||\vec{v}|\cos\theta$ if $0 < \theta < 90^\circ$

$2|\vec{u}||\vec{v}|\cos\theta < -2|\vec{u}||\vec{v}|\cos\theta$ if $90^\circ < \theta < 180^\circ$

since $\cos\theta < 0$

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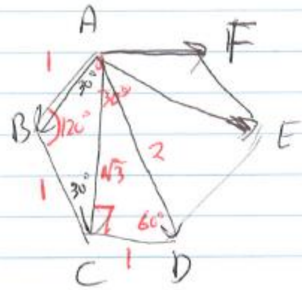
17.



$$\vec{AB}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{AF}$$

$$=\vec{AB}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{CD} \quad (\vec{AF}=\vec{CD})$$

$$=\vec{AB}+\underbrace{\vec{AC}+\vec{CD}}+\vec{AD}+\vec{AE}$$

$$=\vec{AB}+2\vec{AD}+\vec{AE}$$

$$=\vec{ED}+2\vec{AD}+\vec{AE} \quad (\vec{AB}=\vec{ED})$$

$$=2\vec{AD}+\vec{AE}+\vec{ED}$$

$$=3\vec{AD}$$

interior angles of a hexagon sum to $(6-2)180°$
$= 4(180)$
$= 720°$

$\therefore$ each interior angles is $\frac{720}{6}=120°$

$$|\vec{AC}|^2 = 1^2+1^2-2(1)(1)\cos 120°$$

$$|\vec{AC}| = \sqrt{3}$$

$$|\vec{AD}|^2 = 1^2+(\sqrt{3})^2$$

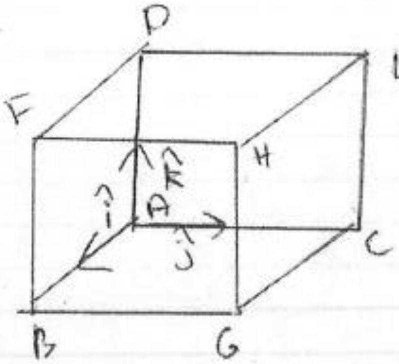
$$|\vec{AD}| = 2 \qquad \therefore 3|\vec{AD}| = 2(3)$$
$$= 6 \text{ units}$$

$\therefore \vec{AB}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{AF} = 3\vec{AD}$

has a magnitude of 3 and makes an angle of 60° with AB

20.

$\hat{i}, \hat{j}, \hat{k}$ are unit vectors (magnitude of 1)



a) $\vec{FG} = \vec{FB} + \vec{BG}$ (triangle law)

$= -\hat{k} + \hat{j}$

$= \hat{j} - \hat{k}$

b) <u>top</u> $\vec{DH} = \vec{DF} + \vec{FH}$

$= \hat{i} + \hat{j}$

<u>bottom</u> $\vec{AG} = \vec{DH} = \hat{i} + \hat{j}$

<u>right</u> $\vec{HC} = \vec{HG} + \vec{GC}$ <u>or</u> $\vec{CH} = \hat{k} + \hat{i}$

$= -\hat{k} + -\hat{i}$ $\quad \vec{CH} = -\vec{HC}$.

$= -\hat{k} - \hat{i}$

– either is fine –

(4.2)

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21. Prove that

$$|\vec{u}+\vec{v}|^2 + |\vec{u}-\vec{v}|^2 = 2(|\vec{u}|^2 + |\vec{v}|^2).$$

$\boxed{\text{geometric proof}}$ $\Leftarrow$ from cosine law.

$$|\vec{u}+\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + 2|\vec{u}||\vec{v}|\cos\theta.$$

$$|\vec{u}-\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|\cos\theta$$

$$\therefore LS = |\vec{u}+\vec{v}|^2 + |\vec{u}-\vec{v}|^2$$

$$= |\vec{u}|^2 + |\vec{v}|^2 + 2|\vec{u}||\vec{v}|\cos\theta + |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|\cos\theta$$

$$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{u}|^2 + |\vec{v}|^2$$

$$= 2(|\vec{u}|^2 + |\vec{v}|^2).$$

$$= RS \qquad \square.$$