Section 5.4

1. $\vec{w} = \vec{u} \times \vec{v}$ so $\vec{w}$ is $\perp$ to $\vec{u}$ and $\vec{v}$.

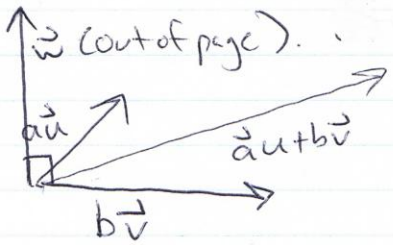
Means that $\vec{w}$ is not on the same plane as $\vec{u}$ and $\vec{v}$.

$\vec{w} \cdot \vec{u} = 0 \quad (\vec{w} \perp \vec{u})$

$\vec{w} \cdot \vec{v} = 0 \quad (\vec{w} \perp \vec{v})$

$a\vec{u} + b\vec{v}$ is a new vector on the same plane as $\vec{u}$ and $\vec{v}$. $\vec{w}$ is $\perp$ to any vector in this plane

$\vec{w} \cdot (a\vec{u} + b\vec{v}) = 0$.



5. $\vec{a} = (4, -3, 1) \quad \vec{b} = (2, 3, -1)$

let $\vec{v} = \vec{a} \times \vec{b} = (0, 6, 18)$

normalize $\vec{v}$

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{(0,6,18)}{\sqrt{0^2+6^2+18^2}} = (0, \tfrac{6}{\sqrt{360}}, \tfrac{18}{\sqrt{360}})$$

$$= (0, \tfrac{6}{6\sqrt{10}}, \tfrac{18}{6\sqrt{10}}) = (0, \tfrac{\sqrt{10}}{10}, \tfrac{3\sqrt{10}}{10})$$

text leaves as $(0, \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}})$ boo!!

* also $(0, -\frac{\sqrt{10}}{10}, -\frac{3\sqrt{10}}{10})$ is a solution!

Section 5.4

p. 186 8. Let $\vec{u} = (u_1, u_2, u_3)$ $\vec{v} = (v_1, v_2, v_3)$

a) Prove $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$

$LS = \vec{u} \times \vec{v}$

$= (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$

$RS = -\vec{v} \times \vec{u}$

$= (-v_1, -v_2, -v_3) \times (u_1, u_2, u_3)$

$= (-u_3v_2 + u_2v_3, -u_1v_3 + u_3v_1, -u_2v_1 + u_1v_2)$

$= (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$

$= LS$

□.

b) if $\vec{u}$ and $\vec{v}$ are collinear then $\vec{u} = k\vec{v}$ for some $k \in \mathbb{R}$

$\vec{u} = k\vec{v}$

$\vec{u} = (kv_1, kv_2, kv_3)$

$\vec{u} \times \vec{v} = (kv_1, kv_2, kv_3) \times (v_1, v_2, v_3)$

$= (kv_2v_3 - kv_2v_3, kv_3v_1 - kv_3v_1, kv_1v_2 - kv_1v_2)$

$= (0, 0, 0) = \vec{0}$. Easier proof geometrically

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9. Prove that $|\vec{a} \times \vec{b}| = \sqrt{(\vec{a}\cdot\vec{a})(\vec{b}\cdot\vec{b}) - (\vec{a}\cdot\vec{b})^2}$

$RS = \sqrt{|\vec{a}|^2|\vec{b}|^2 - (|\vec{a}||\vec{b}|\cos\theta)^2}$

$= \sqrt{|\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta}$

$= \sqrt{|\vec{a}|^2|\vec{b}|^2(1-\cos^2\theta)}$

$= \sqrt{|\vec{a}|^2|\vec{b}|^2\sin^2\theta}$

$= |\vec{a}|\,|\vec{b}|\sin\theta$

$= |\vec{a} \times \vec{b}|$

□.