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2. Proj of $\vec{u}$ onto $\vec{v}$

a) $= \dfrac{(\vec{u}\cdot\vec{v})\vec{v}}{|\vec{v}|^2}$

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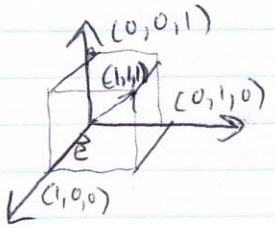
if $\vec{u}, \vec{v}$ are non-zero then $\vec{u}\cdot\vec{v} = 0$

$\vec{u}$ and $\vec{v}$ are $\perp$.

b) Yes. In either case $\vec{u}\cdot\vec{v} = 0$, so projection is zero.

5.

a)



edge, $\vec{e} = (1,0,0)$

body diagonal $= \vec{d} = (1,1,1)$

Proj of $\vec{e}$ onto $\vec{d}$ $= \dfrac{(\vec{e}\cdot\vec{d})\vec{d}}{|\vec{d}|^2}$

$= \dfrac{(1,0,0)\cdot(1,1,1)\ (1,1,1)}{1^2+1^2+1^2}$

$= \dfrac{(1,1,1)}{3} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

b) Proj of $\vec{d}$ onto $\vec{e}$ = $\frac{(\vec{d}\cdot\vec{e})\vec{e}}{|\vec{e}|^2}$

$= \frac{(1,1,1)\cdot(1,0,0)\ (1,0,0)}{1^2}$

$= (1,0,0)$.

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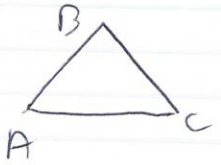
7. a) A(7,3,4) B(1,0,6) and C(4,5,-2)



Can use vectors $\vec{AB}$ and $\vec{AC}$.

Or $\vec{BA}$ and $\vec{BC}$ or $\vec{CA}$ and $\vec{CB}$

(vectors must be tail-to-tail)

$\vec{AB} = (-6,-3,2)$ $\vec{AC} = (-3,2,-6)$

$$\text{Area} = \frac{|\vec{AB} \times \vec{AC}|}{2}$$

$$= \frac{|(14,-42,-21)|}{2}$$

$$= \frac{\sqrt{14^2+(-42)^2+(-21)^2}}{2}$$

$$= \frac{49}{2}$$

16. $W = \vec{F} \cdot \vec{d}$

$\vec{d} = \vec{PQ} = (7,5)$

$|\vec{F}| = 10$ dir of vector $\vec{v} = (1,1)$

$$\hat{v} = \frac{(1,1)}{\sqrt{1^2+1^2}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$\hat{v}$ is unit vector in direction of $\vec{F}$

$\therefore \vec{F} = 10\hat{v}$

$$= \left(\frac{10\sqrt{2}}{2}, \frac{10\sqrt{2}}{2}\right) = (5\sqrt{2}, 5\sqrt{2})$$

$$W = (5\sqrt{2}, 5\sqrt{2}) \cdot (7,5)$$

$$= 35\sqrt{2} + 25\sqrt{2}$$

$$= 60\sqrt{2}$$

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or $\vec{d} = (7,5)$ $\quad |\vec{d}| = \sqrt{7^2+5^2} = \sqrt{74}$

$W = \vec{F} \cdot \vec{d}$ $\quad |\vec{F}| = 10$

$= |\vec{F}||\vec{d}| \cos\theta$ $\quad \cos\theta = \dfrac{\vec{v} \cdot \vec{d}}{|\vec{v}||\vec{d}|}$ since $\vec{v}$ and $\vec{F}$ have same direction

$= (10)(\sqrt{74})\dfrac{12}{\sqrt{148}}$ $\quad \cos\theta = \dfrac{(1,1)\cdot(7,5)}{\sqrt{2}\sqrt{74}} = \dfrac{12}{\sqrt{148}}$

$= 60\sqrt{2}$

17. $W = \vec{F} \cdot \vec{d}$ (similar to #16)
some steps skipped.

$|\vec{F}| = 30$

$\vec{d} = \vec{AB} = \langle 1, -2, -3 \rangle$

dir of force same as $\vec{v} = \langle -2, 1, 5 \rangle$

$$\hat{v} = \frac{\langle -2, 1, 5 \rangle}{\sqrt{(-2)^2 + 1^2 + 5^2}}$$

$$= \frac{\langle -2, 1, 5 \rangle}{\sqrt{30}} = \left\langle \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}} \right\rangle$$

$$= \left\langle \frac{-\sqrt{30}}{15}, \frac{\sqrt{30}}{30}, \frac{\sqrt{30}}{6} \right\rangle$$

so $\vec{F} = 30\hat{v}$

$$= \langle -2\sqrt{30}, \sqrt{30}, 5\sqrt{30} \rangle$$

$$W = \vec{F} \cdot \vec{d}$$

$$= -2\sqrt{30} - 2\sqrt{30} - 15\sqrt{30}$$

$$= -19\sqrt{30}$$

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or find angle between $\langle -2, 1, 5 \rangle$ and $\langle 1, -2, -3 \rangle$
then use $W = |\vec{F}||\vec{d}| \cos\theta$.

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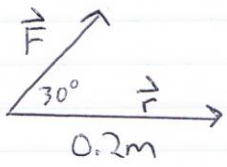
a) 

$|\vec{r} \times \vec{F}| = 50(0.2)\sin 30°$

$= 5\,N\cdot m$

torque is $5\,N\cdot m$ (direction not relevant).

b) Achieve max. torque when $\theta = 90°$

since $|\vec{r} \times \vec{F}| = 50(0.2)\sin\theta$ $\quad \sin\theta = 1$ (max) at $\theta = 90°$

$= 50(0.2)1$

$= 10\,N\cdot m$