Chapter 5 Review Exercise

7. Let $\vec{a} = (6, 3, -2)$ $\vec{b} = (-2, p, -4)$

$$\vec{a} \cdot \vec{b} = -12 + 3p + 8 = -4 + 3p$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$\frac{4}{21} = \frac{-4+3p}{\sqrt{6^2+3^2+(-2)^2}\,\sqrt{(-2)^2+p^2+(-4)^2}}$$

$$\frac{4}{21} = \frac{-4+3p}{(7)\sqrt{20+p^2}}$$

$$28\sqrt{20+p^2} = -84+63p$$

square sides: $784(20+p^2) = (-84+63p)^2$

$$15680 + 784p^2 = 3969p^2 - 10584p + 7056$$

$$-3185p^2 + 10584p + 8624 = 0$$

$p = 4$ or $p = -44/65$ (use quadratic formula)

inadmissible. Result of squaring both sides, can add a negative root that doesn't satisfy original equation.

pg 194

8. $\hat{i}+\hat{j}+\hat{k} = (1,1,1)$

$\lambda^2\hat{i} - 2\lambda\hat{j} + \hat{k} \rightarrow (\lambda^2, -2\lambda, 1)$

$(1,1,1)\cdot(\lambda^2,-2\lambda,1) = 0$

$\lambda^2 - 2\lambda + 1 = 0$

$(\lambda-1)^2 = 0$

$\lambda = 1.$

9. $(4\vec{x}-\vec{y})\cdot(2\vec{x}+3\vec{y})$

$= 8|\vec{x}|^2 + 10\,\vec{x}\cdot\vec{y} - 3|\vec{y}|^2$

$= 8|\vec{x}|^2 + 10|\vec{x}||\vec{y}|\cos\theta - 3|\vec{y}|^2$

$= 8(3)^2 + 10(3)(4)\cos 60^\circ - 3(4)^2$

$= 84.$

10. If $\vec{u}$ has dir. angles $\alpha_1, \beta_1, \gamma_1$ then

Unit vector: $\hat{u} = (\cos\alpha_1, \cos\beta_1, \cos\gamma_1)$.

Also $\hat{v} = (\cos\alpha_2, \cos\beta_2, \cos\gamma_2)$

$\hat{u}\cdot\hat{v} = 0$ since $\vec{u}, \vec{v}$ are $\perp$

$(\cos\alpha_1, \cos\beta_1, \cos\gamma_1)\cdot(\cos\alpha_2, \cos\beta_2, \cos\gamma_2) = 0$

$\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2 = 0$

$\square$

11. $\vec{x} \cdot \vec{y} = \frac{1}{2}(|\vec{x}+\vec{y}|^2 - |\vec{x}|^2 - |\vec{y}|^2)$.

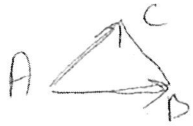
$$Rs = \frac{1}{2}\left(|\vec{x}|^2 + |\vec{y}|^2 + 2|\vec{x}||\vec{y}|\cos\theta - |\vec{x}|^2 - |\vec{y}|^2\right)$$

$$= \frac{1}{2}\left(2|\vec{x}||\vec{y}|\cos\theta\right)$$

$$= |\vec{x}||\vec{y}|\cos\theta$$

$$= \vec{x} \cdot \vec{y}$$

12. $A(-1,3,4)$ $B(3,-1,1)$ $C(5,1,1)$.



$\vec{AB} = (4,-4,-3)$ $\vec{AC} = (6,-2,-3)$

a) $\vec{AB} \cdot \vec{AC} = 24 + 8 + 6 = 38.$

$\vec{CB} = (2,2,0)$ $\qquad \vec{CB} \cdot \vec{AB} = 8 - 8 = 0.$

$\therefore$ $AB \perp CB$. right triangle.

b) $$\text{Area} = \frac{|\vec{AB} \times \vec{AC}|}{2}$$

$$= \frac{|(6,-6,16)|}{2}$$

$$= \frac{\sqrt{328}}{2}$$

$$= \frac{2\sqrt{82}}{2}$$

$= \sqrt{82}$ square units

c) $\vec{CB} = (-2,-2,0)$

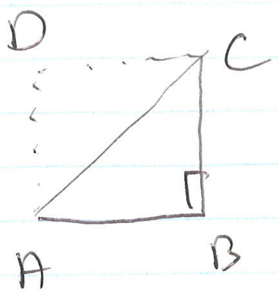
$P = |\vec{AB}| + |\vec{AC}| + |\vec{CB}|$

$P = \sqrt{4^2+(-4)^2+(-3)^2} + \sqrt{6^2+(-2)^2+(-3)^2} + \sqrt{(-2)^2+(-2)^2}$

$P = \sqrt{41} + \sqrt{49} + \sqrt{8} \rightarrow P \doteq 16.2$

Pg 195 12d)

d) we know $AB \perp CB$.



Find $D(x, y, z)$

$\overrightarrow{AB} = \overrightarrow{DC}$

$(4, -4, -3) = (5-x, 1-y, 1-z)$

$\therefore \quad 4 = 5 - x \qquad -4 = 1 - y \qquad -3 = 1 - z$

$x = 1 \qquad y = 5 \qquad z = 4$

D is $(1, 5, 4)$.

16. a) $\vec{a} \times \vec{b}$ is $\perp$ to $\vec{a}$ and $\vec{b}$

(outside of plane of $\vec{a}, \vec{b}$).

Imagine $\vec{a} \times \vec{b}$ coming out of the page and $\vec{a}, \vec{b}$ on the page.

$\therefore$ Any vector $\perp$ to $\vec{a} \times \vec{b}$ must be on this page with $\vec{a}$ and $\vec{b}$,

$\therefore (\vec{a} \times \vec{b}) \times \vec{c}$ lies in plane of $\vec{a}$ and $\vec{b}$.