Section 5.3

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5.

a) 3 vectors $\perp$ to $(2,-3)$

any vector (a,b) such that $(a,b)\cdot(2,-3)=0$

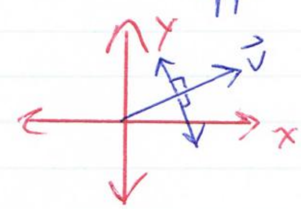
$(a,b)\cdot(2,-3)=0$

$2a-3b=0$ or $2a=3b$.

lots of possible vectors.

$(6,4)$, $(-6,-4)$ and $(15,10)$ are some examples

b) Only 2. One has opposite direction of the other.



6.

a) Any vector such that

$(a,b,c)\cdot(2,-3,1)=0$

$2a-3b+c=0$

<u>examples</u> $(1,1,1)$ $(0,1,3)$ $(2,0,-4)$.

b) infinite. A vector in space has a lot of unit vectors $\perp$ to it.

also out of page, into page etc.

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9. $\vec{a} = (2,3,7)$ $\vec{b} = (-4, y, -14)$

a) If collinear then $\vec{a} = k\vec{b}$ for some $k \in R$.

$(2,3,7) = k(-4, y, -14)$

$2 = -4k \qquad 3 = ky$

$k = -\frac{1}{2} \qquad 3 = -\frac{1}{2}y$

$\boxed{y = -6}$

b) $\vec{a} \cdot \vec{b} = 0$

$(2,3,7) \cdot (-4, y, -14) = 0$

$-8 + 3y - 98 = 0$

$3y - 106 = 0$

$3y = 106$

$y = \frac{106}{3}$

11. $\vec{a} \cdot \vec{b} = 0$

$(2,3,4) \cdot (10, y, z) = 0$

$20 + 3y + 4z = 0$

or $3y + 4z = -20$ that's how they are related.

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14. a) $(4\hat{i}-\hat{j})\cdot\hat{j}$

$= 4\hat{i}\cdot\hat{j} - |\hat{j}|^2$

$= 4(0) - (1)^2$

$= -1$

$\hat{i}$ and $\hat{j}$ are $\perp$

$\therefore \hat{i}\cdot\hat{j} = 0$

c) $(\hat{i}-4\hat{k})\circ(\hat{i}-4\hat{k})$

$= |\hat{i}|^2 - 4\hat{i}\cdot\hat{k} - 4\hat{i}\cdot\hat{k} + 16|\hat{k}|^2$

$= 1 - 8\hat{i}\cdot\hat{k} + 16(1)$

$= 1 - 0 + 16$

$= 17.$

17. $(2\vec{a}+\vec{b})\cdot(\vec{a}-3\vec{b}) = 0$ (since they are $\perp$)

$2|\vec{a}|^2 - 5\vec{a}\cdot\vec{b} - 3|\vec{b}|^2 = 0$

but $|\vec{a}| = 2|\vec{b}|$

$2(2|\vec{b}|)^2 - 5|\vec{a}||\vec{b}|\cos\theta - 3|\vec{b}|^2 = 0$

$5|\vec{b}|^2 - 5|\vec{a}||\vec{b}|\cos\theta = 0$

$5|\vec{b}|^2 - 5(2)|\vec{b}||\vec{b}|\cos\theta = 0$

$5|\vec{b}|^2 - 10|\vec{b}|^2\cos\theta = 0$

$5|\vec{b}|^2[1 - 2\cos\theta] = 0$ (factoring).

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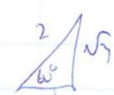
$$1 - 2\cos\theta = 0$$
$$2\cos\theta = 1$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = 60^\circ.$$

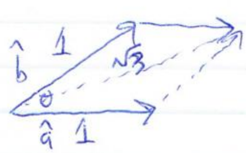
18. a) $(6\hat{a}+\hat{b})\cdot(\hat{a}-2\hat{b})$

$$= 6|\hat{a}|^2 - 11\,\hat{a}\cdot\hat{b} - 2|\hat{b}|^2$$
$$= 6(1)^2 - 11\,|\hat{a}||\hat{b}|\cos\theta - 2(1)$$



$$= 4 - 11(1)(1)\cos 60^\circ$$
$$= 4 - 11\left(\tfrac{1}{2}\right)$$
$$= \frac{8}{2} - \frac{11}{2}$$
$$= -\frac{3}{2}.$$

b) $|\hat{a}+\hat{b}| = \sqrt{3}$



$$|\hat{a}+\hat{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$
$$(\sqrt{3})^2 = 1^2 + 1^2 + 2(1)(1)\cos\theta$$
$$\cos\theta = \frac{(\sqrt{3})^2 - 1 - 1}{2}$$

$\cos\theta = \frac{1}{2}$ $\quad\theta = 60^\circ$ again!

$(2\hat{a}-5\hat{b})\cdot(\hat{b}+3\hat{a})$

$= -13\,\hat{a}\cdot\hat{b} + 1$

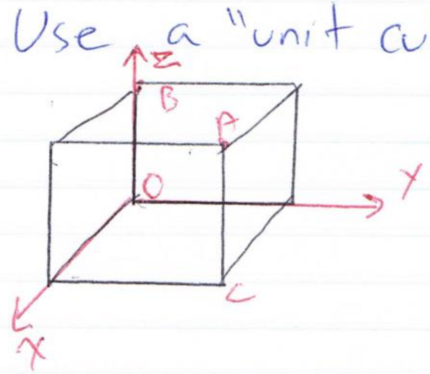
$= -13(1)(1)\cos\theta + 1$

$= -13\,(1)(1)\cos 60^\circ + 1$

$= -\frac{13}{2} + 1$

$= -11/2$

24. Use a "unit cube"



A(1,1,1)
C(1,1,0)
B(0,0,1)

diagonals are $\vec{OA} = (1,1,1)$ and $\vec{BC} = (1,1,-1)$
(other possibilitres as well)

$\cos\theta = \dfrac{\vec{OA}\cdot\vec{BC}}{|\vec{OA}|\,|\vec{BC}|}$

$\cos\theta = \dfrac{1}{\sqrt{3}\,\sqrt{3}}$

$\cos\theta = 1/3$

$\theta \doteq 71^\circ$

Note that $180 - 71^\circ$
$= 109^\circ$ also possible