MCV4U

Vector Addition

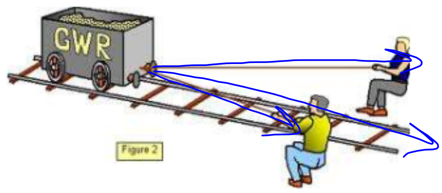
Lesson 2

Last day we introduced the vector as a way of describing quantities that have both magnitude and direction. We defined equality of vectors, the "zero vector" and the negative of a vector.

Today we will examine vector addition. What does it mean to add two vectors together?

Consider two vectors both representing a force as in the picture below.

The sum of these two vectors will represent the combined force acting on the rail car.

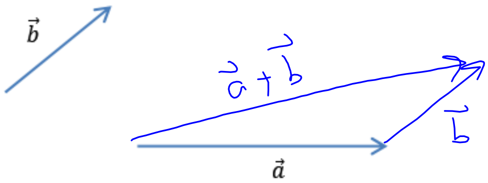


The addition of 2 vectors is calculated two different ways, depending on how the vectors are oriented.

The Triangle Law

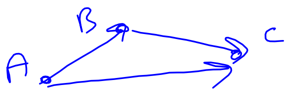
head-to-tail

To find the sum of two vectors $\vec{a}$ and $\vec{b}$ using the triangle law, draw the vectors **head to tail**. The sum $\vec{a}+\vec{b}$ (often called the resultant) is the vector formed from the tail of vector $\vec{a}$ to the head of vector $\vec{b}$.



Think: Can you use the triangle law of vector addition to simplify the following:

$\overrightarrow{AB}+\overrightarrow{BC} = \overrightarrow{AC}$

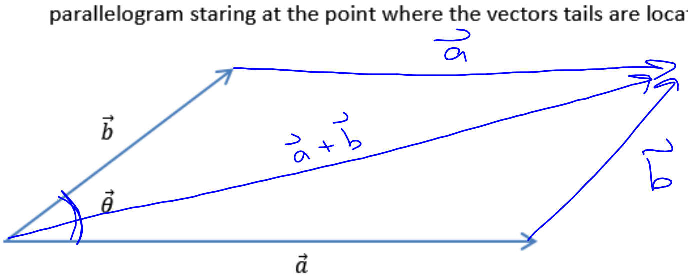


$\overrightarrow{AB}+\overrightarrow{BA} = \overrightarrow{AA}$
$= \vec{0}$

The Parallelogram Law

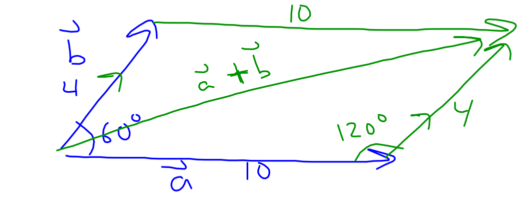
tail-to-tail

To find the sum of two vectors $\vec{a}$ and $\vec{b}$ using the parallelogram law, draw the vectors **tail to tail**. Complete the parallelogram with these 2 vectors as sides. The sum $\vec{a}+\vec{b}$ is the vector formed on the diagonal of the parallelogram staring at the point where the vectors tails are located.



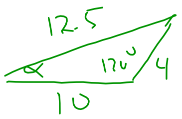
Example

Find $\vec{a}+\vec{b}$ given that $|\vec{a}| = 10$, $|\vec{b}| = 4$ and the angle between the two vectors is 60°.



$$|\vec{a}+\vec{b}|^2 = 10^2 + 4^2 - 2(10)(4)\cos 120^\circ$$

$$|\vec{a}+\vec{b}| \doteq 12.5$$



$$\frac{\sin\alpha}{4} = \frac{\sin 120^\circ}{12.5}$$

$$\sin\alpha = \frac{4\sin 120^\circ}{12.5}$$

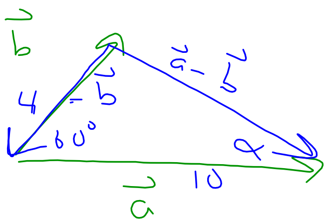
$$\alpha \doteq 16^\circ$$

Vector Subtraction

Can you think of a reasonable definition to subtract two vectors?

Find $\vec{a}-\vec{b}$ for vectors $\vec{a}$ and $\vec{b}$ above.

$$\vec{a} + (-\vec{b}) = -\vec{b} + \vec{a}$$

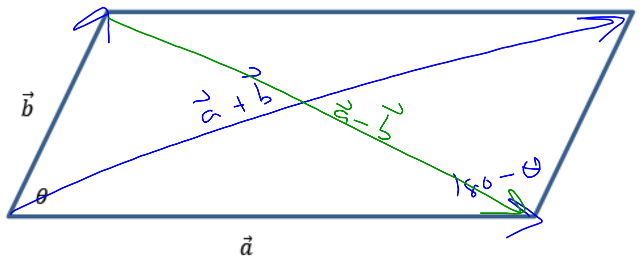


$$|\vec{a}-\vec{b}|^2 = 10^2 + 4^2 - 2(10)(4)\cos 60^\circ$$

$$|\vec{a}-\vec{b}| \doteq 8.7$$

The example above shows us how the magnitude of the resultant of two vectors can always be calculated using the cosine law.

$\vec{a} - \vec{b}$
$= \vec{a} + -\vec{b}$
$= -\vec{b} + \vec{a}$



$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(180-\theta)$ $\quad |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

use either

but $\cos(180-\theta) = -\cos\theta$

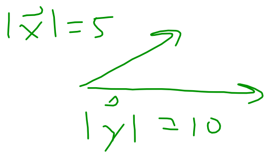
$= |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|(-\cos\theta)$

$= |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$

The direction of the resultant can then be calculated using the sine law.

Triangle Inequality

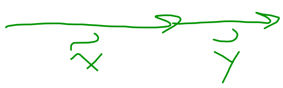
The triangle inequality states that: $|\vec{x} + \vec{y}| \le |\vec{x}| + |\vec{y}|$



$|\vec{x} + \vec{y}| \le 15$

When will $|\vec{x} + \vec{y}| = |\vec{x}| + |\vec{y}|$?

$\theta = 0°$



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