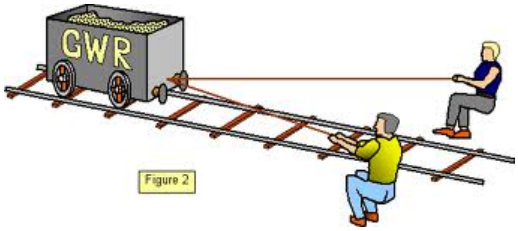
Vector Addition

Last day we introduced the vector as a way of describing quantities that have both magnitude and direction. We defined equality of vectors, the "zero vector" and the negative of a vector.

Today we will examine vector addition. What does it mean to add two vectors together?

Consider two vectors both representing a force as in the picture below.

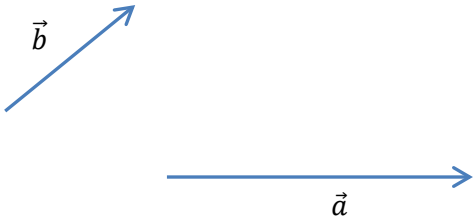
The sum of these two vectors will represent the combined force acting on the rail car.



The addition of 2 vectors is calculated two different ways, depending on how the vectors are oriented.

The Triangle Law

To find the sum of two vectors $\vec{a}$ and $\vec{b}$ using the triangle law, draw the vectors **head to tail**. The sum $\vec{a} + \vec{b}$ (often called the resultant) is the vector formed from the tail of vector $\vec{a}$ to the head of vector $\vec{b}$.

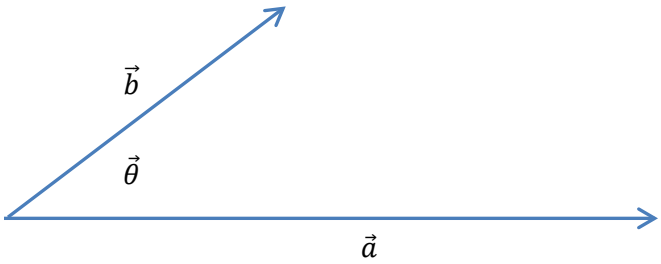


Think: Can you use the triangle law of vector addition to simplify the following:

$\overrightarrow{AB} + \overrightarrow{BC}$

The Parallelogram Law

To find the sum of two vectors $\vec{a}$ and $\vec{b}$ using the parallelogram law, draw the vectors **tail to tail**. Complete the parallelogram with these 2 vectors as sides. The sum $\vec{a} + \vec{b}$ is the vector formed on the diagonal of the parallelogram staring at the point where the vectors tails are located.



Example

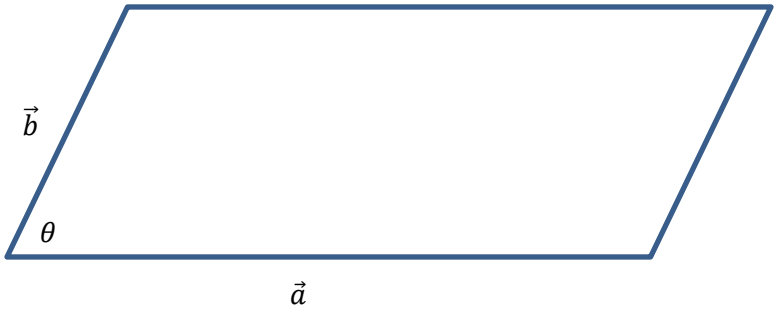
Find $\vec{a} + \vec{b}$ given that $|\vec{a}| = 10$, $\left|\vec{b}\right| = 4$ and the angle between the two vectors is 60°.

Vector Subtraction

Can you think of a reasonable definition to subtract two vectors?

Find $\vec{a} - \vec{b}$ for vectors $\vec{a}$ and $\vec{b}$ above.

The example above shows us how the magnitude of the resultant of two vectors can always be calculated using the cosine law.



$|\vec{a}+\vec{b}|^2 =$ $|\vec{a}-\vec{b}|^2 =$

The direction of the resultant can then be calculated using the sine law.

Triangle Inequality

Show (geometrically) that the following will always hold: $|\vec{x}+\vec{y}| \leq |\vec{x}| + |\vec{y}|$ (the triangle inequality)

When will $|\vec{x}+\vec{y}| = |\vec{x}| + |\vec{y}|$?

Text page 133 #1, 3a, 4, 5, 7, 20