MCV4U Lesson 3

Properties of Vector Addition and Scalar Multiplication

Today we will look at some basic properties of vector addition and scalar multiplication and find that these basic laws are similar to those of arithmetic and basic algebra (i.e. they will no surprise you!). However, these existence (or lack of) these properties is examined in linear algebra and provides a basis for an important concept called a "Vector Space" – something many students will study in first year University.

Properties of Vector Addition

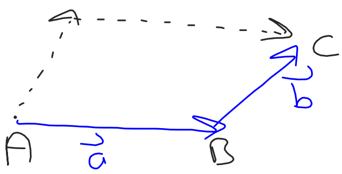
Let $\vec{a}, \vec{b}$ and $\vec{c}$ be vectors. The following properties hold:

Commutative Property- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

Associative Property $\overrightarrow{(a + b)} + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

Zero Vector $\vec{a} + \vec{0} = \vec{a}$ and $\vec{a} + (-\vec{a}) = \vec{0}$

Proof of the Commutative Property: Prove that $\vec{a} + \vec{b} = \vec{b} + \vec{a}$



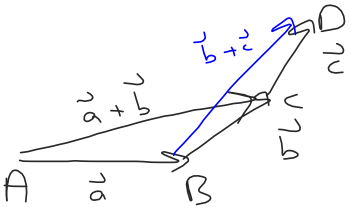
Let $\overrightarrow{AB} = \vec{a}$ Let $\overrightarrow{BC} = \vec{b}$

$LS = \vec{a} + \vec{b}$
$= \overrightarrow{AB} + \overrightarrow{BC}$
$= \overrightarrow{AC}$ ($\triangle$ law)

$RS = \vec{b} + \vec{a}$
$= \overrightarrow{BC} + \overrightarrow{AB}$
$= \overrightarrow{AC}$ (// gram law)

Prove the Associative Property - using the triangle law of addition.

$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$



Let $\vec{a} = \overrightarrow{AB}$, $\vec{b} = \overrightarrow{BC}$, $\vec{c} = \overrightarrow{CD}$

$LS = (\vec{a} + \vec{b}) + \vec{c}$
$= (\overrightarrow{AB} + \overrightarrow{BC}) + \overrightarrow{CD}$
$= \overrightarrow{AC} + \overrightarrow{CD}$ ($\triangle$ law)
$= \overrightarrow{AD}$ ($\triangle$ law)

$RS = \vec{a} + (\vec{b} + \vec{c})$
$= \overrightarrow{AB} + (\overrightarrow{BC} + \overrightarrow{CD})$
$= \overrightarrow{AB} + \overrightarrow{BD}$
$= \overrightarrow{AD}$

$\therefore (\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

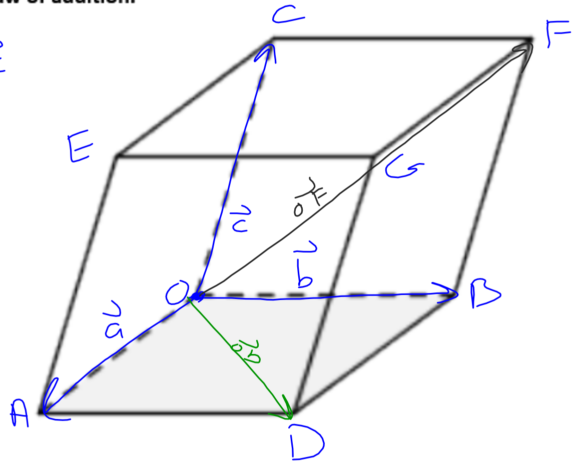
D

3 - space

Prove the Associative Property - using parallelogram law of addition.

$(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$

Let $\vec{a} = \vec{OA}$ $\quad \vec{b} = \vec{OB}$, $\vec{c} = \vec{OC}$



$(\vec{a}+\vec{b})+\vec{c}$

$= (\vec{OA}+\vec{OB})+\vec{OC}$

$= \vec{OD}+\vec{OC}$

$= \vec{OG}$

$\vec{a}+(\vec{b}+\vec{c})$

$= \vec{OA}+(\vec{OB}+\vec{OC})$

$= \vec{OA}+\vec{OF}$

$= \vec{OG}$

Properties of Scalar Multiplication
Let $\vec{a}$ *and* $\vec{b}$ be vectors and m and n be scalars where $m, n \in R$. The following properties hold:
Associative Property $m(n\vec{a}) = (mn)\vec{a}$ $\quad 3(2\vec{a}) = 6\vec{a}$
Distributive Property $(m+n)\vec{a} = m\vec{a}+n\vec{a}$ AND $m(\vec{a}+\vec{b}) = m\vec{a}+m\vec{b}$ $\quad 2(\vec{a}+\vec{b}) = 2\vec{a}+2\vec{b}$

Example: If $\vec{a} = 3\vec{x} - 5\vec{y}$ and $\vec{b} = -2\vec{x} + 9\vec{y}$ then find $5\vec{a} - 9\vec{b}$. (State each property used)

$5\vec{a} - 9\vec{b}$

$= 5(3\vec{x}-5\vec{y}) - 9(-2\vec{x}+9\vec{y})$

$= 15\vec{x} - 25\vec{y} + 18\vec{x} - 81\vec{y}$ $\quad$ distributive, associative

$= 15\vec{x} + 18\vec{x} - 25\vec{y} - 81\vec{y}$ $\quad$ commutative property

$= (15+18)\vec{x} + (-25-81)\vec{y}$ $\quad$ distributive

$= 33\vec{x} - 106\vec{y}$

Homework

1. Simplify the following: $\overrightarrow{AB} - \overrightarrow{DC} + \overrightarrow{BC} - \overrightarrow{ED}$

2. The diagonals of a parallelogram ABCD meet at point E. Show that $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED} = \vec{0}$

3. Text page 133 #3bcd, 6, 10 (state properties used), 18

4. In the trapezoid *OACB*, $\angle AOB = \angle OAC = 60°$ and *OA* and *BC* are parallel. *OB* = *BC* = *CA* = 2 units, and $\hat{x}$ and $\hat{y}$ are both unit vectors (i.e. have a magnitude of 1) as shown. *M* and *N* are the midpoints of *BC* and *CA,* respectively. Express each of the following vectors in terms of $\hat{x}$ and $\hat{y}$.

a) $\overrightarrow{OM}$ b) $\overrightarrow{CO}$

c) $\overrightarrow{OA}$ d) $\overrightarrow{AC}$

e) $\overrightarrow{ON}$