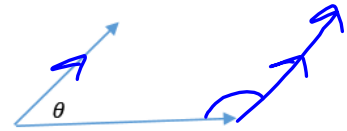1. Remember: **The angle between two vectors is defined as being the angle made when the vectors are placed *tail-to-tail*.**

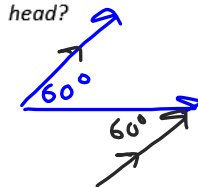   Suppose the angle between two vectors is 60°.

   a) What angle would be made if the same vectors were moved so that they were *head-to-tail*?

   $$\theta = 180° - 60°$$
   $$\theta = 120°$$

   b) What angle would be made if the same vectors were moved so that they were *head-to-head*?

   $60°$ $60°$ always equal.

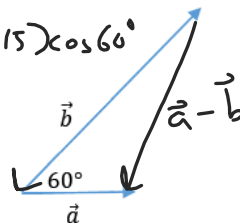2. Two vectors $\vec{a}$ and $\vec{b}$ are shown in the diagram below. $|\vec{a}| = 3$ and $|\vec{b}| = 15$.

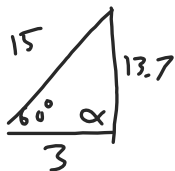   a) Find $|\vec{a} - \vec{b}|$

   $\boxed{\vec{a} - \vec{b} = -\vec{b} + \vec{a}}$

   $$|\vec{a} - \vec{b}|^2 = 3^2 + 15^2 - 2(3)(15)\cos 60°$$
   $$\doteq 13.7$$

   b) Find the direction $\vec{a} - \vec{b}$ makes with vector $\vec{a}$

   $$\frac{\sin\alpha}{15} = \frac{\sin 60°}{13.7}$$
   $$\sin\alpha = \frac{15\sin 60°}{13.7}$$
   $$\alpha \doteq 71°$$

   ∴ $\vec{a} - \vec{b}$ makes an angle of 71° with $\vec{a}$?

   $$\cos\alpha = \frac{3^2 + 13.7^2 - 15^2}{2(3)(13.7)}$$
   $$\alpha \doteq 110° \checkmark$$

3. There may have been two solutions for the angle in 2b) above. Which one is correct? We will examine why below.

$$109° - 110°$$

There is never ambiguity when adding/subtracting vectors. However, you must be careful with your use of the Sine Law. There are always two solutions between 0° and 180° when taking the inverse sine. Make sure you know you are getting the correct one!

Velocity is an example of a vector (speed + direction). Velocity is always relative. (ie. Relative to the ground, to moving water, to the air, to another moving object).

Example An airplane steering N45°W (North then 45° West) at 500 km/h (relative to the air) encounters a wind of 120km/h **from** N65⁰E. Determine the velocity of the plane relative to the ground.

$$\vec{V}_{ground} = \vec{V}_{air} + \vec{w}$$

$$|\vec{w}| = 120 \qquad |\vec{V}_{air}| = 500$$
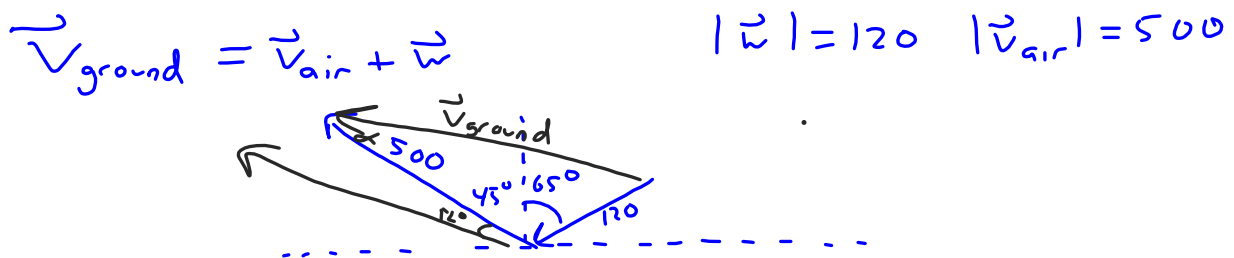
$$|\vec{V}_{ground}|^2 = 500^2 + 120^2 - 2(500)(120)\cos 110°$$

$$|\vec{V}_{ground}| \doteq 553 \, \text{km/h}$$

$$\frac{\sin \alpha}{120} = \frac{\sin 110°}{553}$$

$$\alpha \doteq 12°$$

direction is N57°W

or W33°N

or 33° North of West.

1. Text page 150 #7, 8, 10, 13 and page 153 #9, 10, 15

2. Three distinct **unit vectors** $\vec{a}, \vec{b}$ and $\vec{c}$ lie in the same plane (i.e. they can all be drawn on the same flat surface). The angle between $\vec{a}$ and $\vec{b}$ is 45°. The angle between $\vec{b}$ and $\vec{c}$ is also 45°. Draw a picture of the vector $\vec{a} - 2\vec{b} + 3\vec{c}$ and calculate the magnitude of this vector. (ANSWER – appox. 1.64)