# Algebraic Vectors

So far we have looked at vectors geometrically. There are many issues and challenges with this representation. Today we will look at vectors algebraically, by putting them on the Cartesian plane and examining their "components".

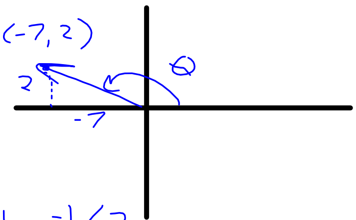
## 2-space ($R^2$)

Let $\vec{v}$ be a vector in $R^2$ (2-space). The vector $\vec{v}$ can be plotted on the Cartesian plane such that the tail of $\vec{v}$ is at the origin and the head of $\vec{v}$ falls at some point with coordinates *(a,b)*. $\vec{v}$ can now be described by the point (a, b). When described this way the vector is known as an **algebraic vector.** *a* and *b* are the **components** (or sometimes vector numbers) of this vector.

Let *P* be point (a,b). We often refer to vector $\overrightarrow{OP}$ when defining point-to-point vectors.

**Example**

Find the magnitude and direction of vector $\vec{a} = (-7, 2)$.




$$|\vec{a}|^2 = (-7)^2 + 2^2$$

$$|\vec{a}| = \sqrt{53}$$

$$\theta = \tan^{-1}\left(-\frac{2}{7}\right)$$

$$\theta \doteq 164°$$

**In general:**

$$\vec{v} = (a, b) \qquad |\vec{v}| = \sqrt{a^2 + b^2} \qquad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

angle with positive x-axis

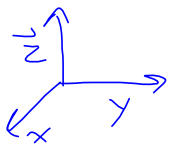
## 3-space ($R^3$)

In order to represent vectors in 3-space we must introduce a third coordinate axis. We will use *x, y* and *z.*

We consider *x* to be travelling forward "out of the page" (+) or travelling back (-) "into the page.

*y* travels horizontally (Similar to *x* in Cartesian plane) and *z* travels up and down.

***Right-hand-rule.***

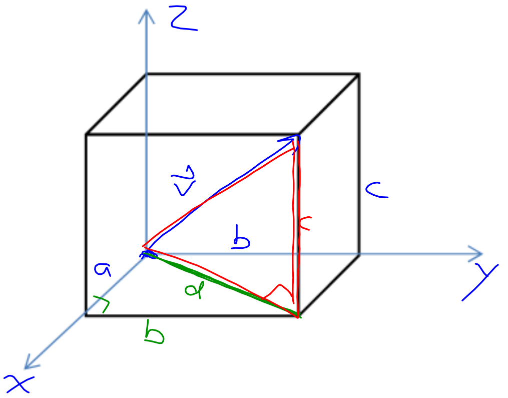



Let $\vec{v}$ be a vector in $R^3$ (3-space). Position $\vec{v}$ so that its tail is on the origin. The head of $\vec{v}$ will fall on some point *(a,b,c)*. We refer to *(a,b,c)* as an algebraic vector in $R^3$.

**How can we calculate the magnitude and direction of a vector $\vec{v} = (a, b, c)$ in $R^3$?**

**Magnitude:**

Below is a rectangular prism draw with one corner at the origin and the opposite corner at (a,b,c). Therefore $\vec{v}$ is a body diagonal of this prism.




$$d = \sqrt{a^2 + b^2}$$

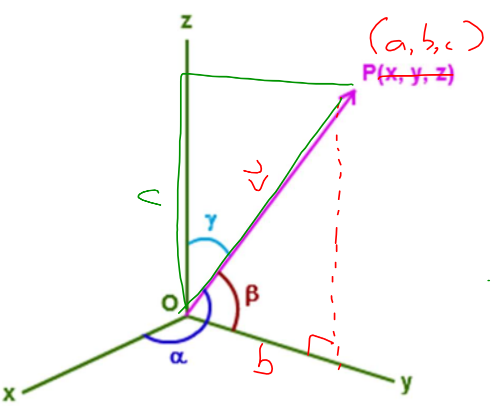
$$|\vec{v}|^2 = \left(\sqrt{a^2+b^2}\right)^2 + c^2$$

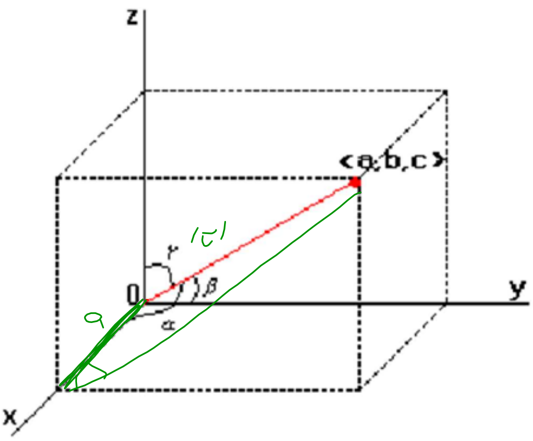
$$|\vec{v}|^2 = a^2 + b^2 + c^2$$

$$|\vec{v}| = \sqrt{a^2+b^2+c^2}$$

**Direction**

The direction of a vector in 3-space is described by finding the measure of the angles it makes with the **positive** x, y and z axes. These angles are named $\alpha$ (alpha), $\beta$ (beta) and $\gamma$ (gamma) respectively. These angles always form a right triangle with the vector being the hypotenuse and with the respective coordinate axis being the adjacent side.




$\cos\alpha = \dfrac{a}{|\vec{v}|}$ $\cos\beta = \dfrac{b}{|\vec{v}|}$ $\cos\gamma = \dfrac{c}{|\vec{v}|}$

$\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called direction cosines.

$\alpha$, $\beta$ and $\gamma$ are direction angles.

A special relationship exists between the direction cosines.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

**Example**

Let $\vec{v} = (3,5,-4)$. Describe this vector by its geometric characteristics. Then verify the direction cosines do in fact have a sum of 1.

$$|\vec{v}| = \sqrt{3^2+5^2+(-4)^2}$$

$$|\vec{v}| = \sqrt{50}$$

$$|\vec{v}| = 5\sqrt{2}$$

$$\cos\alpha = \frac{3}{5\sqrt{2}}, \alpha \doteq 65^\circ$$

$$\cos\beta = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}, \beta = 45^\circ$$

$$\cos\gamma = \frac{-4}{5\sqrt{2}}, \gamma \doteq 124^\circ$$

$$\left(\frac{3}{5\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{-4}{5\sqrt{2}}\right)^2$$

$$= \frac{9}{50} + \frac{1}{2} + \frac{16}{50}$$

$$= \frac{9}{50} + \frac{25}{50} + \frac{16}{50}$$

$$= 1$$

**Text page 166 #1, 6 (include sketch, use exact values), 7,8,11,13ad, #14 (also find direction angles)**