# Operations with Algebraic Vectors

**Most of the operations for algebraic vectors below are defined for vectors in $R^3$. It should be simple to apply these definitions to vectors in $R^2$ as well.**

**Vector Equality** - Let $\vec{u} = (a_1, b_1, c_1)$ and $\vec{v} = (a_2, b_2, c_2)$. We say that $\vec{u} = \vec{v}$ if and only if

**Scalar Multiplication**

Let $\vec{v} = (a, b, c)$ be any vector in $R^3$. Given scalar $k \epsilon R$ then:

$k\vec{v} =$

**Vector Addition/Subtraction**

How do we add two algebraic vectors?

Let $\vec{u} = (a_1, b_1, c_1)$ and $\vec{v} = (a_2, b_2, c_2)$

$\vec{u} + \vec{v} =$

Example: Given $\vec{u} = (3,1,0)$ and $\vec{v} = (1, -1, 3)$ Find $3\vec{u} + 4\vec{v}$:

**Vector Joining 2 Points**

Often we want to define an algebraic vector that goes through to points.

In $R^2$: The point-to-point vector $\overrightarrow{PQ}$ for points P($a_1$, $b_1$) and Q($a_2$,$b_2$) is given by:

In $R^3$: The point-to-point vector $\overrightarrow{PQ}$ for points P($a_1$, $b_1$, $c_1$) with Q($a_2$,$b_2$, $c_2$) is given by:

**Parallel Vectors**

Two vectors $\vec{u}$ and $\vec{v}$ are parallel if and only if:

**Example:** Determine whether $\overrightarrow{AB}||\overrightarrow{CD}$ given A(2,0), B(3,6), C(3, 1) and D(5,-5).

We call parallel vectors "collinear". Collinear refers to the fact that they can be drawn on the same line.

**Collinear**

Three points, A, B and C are collinear if and only if $\overrightarrow{AB} = k\overrightarrow{BC}$ for some $k \in R$.

**Example:** Determine whether or not the points A(9, -6, 15), B(-3, 2, -5) and C(6, -4, 10) are collinear.

## Unit Vectors

A unit vector has a magnitude of one unit. It is denoted by the ^ symbol.

Any vector $\vec{v}$, can be expressed as a scalar product of its magnitude and a unit vector having the same direction.

$\vec{v} =$

**Example**: given $\vec{v} = (4,3)$:

We can rearrange this definition of a vector to derive a **definition of a unit vector.**

So we can use the above process to make a vector into a unit vector having the same direction. We call this **normalizing a vector**.

Example: Normalize the vector $\vec{v} = (2, -3, -6)$

So if we travel one unit from the origin along the vector $\vec{v}$, we will be at point:

Note that the components of a unit vector v, are also the direction cosines for any vector that is a multiple of v! This tells us that:

Use algebraic vectors (i.e. components) to prove the distributive law for vector addition/scalar multiplication. i.e. prove that $k(\vec{a}+\vec{b}) = k\vec{a} + k\vec{b}$

**Unit Vector Notation**

We define 3 standard unit vectors in the direction of x, y and z axes.

In $R^2$:

In $R^3$:

Any algebraic vector can be defined in either component form or in unit vector notation.

**Examples**

(4, -8)

(1, -2, 7)

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