MCV4U

The Dot Product

Today we will define a new operation between 2 vectors – called the dot product. Unlike previous operations with vectors this operation is used for vectors and does not have a similar operation with scalars.

The Dot Product geometric

The *dot product* of two vectors $\vec{u}$ and $\vec{v}$ is given by:

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$

where θ is the angle between the 2 vectors (tail-to-tail).

The dot product is sometimes called the ***scalar product*** or ***an inner product***.

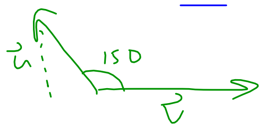
Example Find $\vec{u} \cdot \vec{v}$ if:

a) $|\vec{u}| = 7$, $|\vec{v}| = 15$ and $\theta = 60°$.

$$\vec{u} \cdot \vec{v} = 7(15)\cos 60^\circ$$
$$= 7(15)\left(\tfrac{1}{2}\right)$$
$$= \frac{105}{2}$$

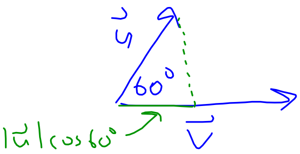
b) $|\vec{u}| = 13$, $|\vec{v}| = 55$ and $\theta = 150°$.

$$\vec{u} \cdot \vec{v} = (13)(55)\cos 150^\circ$$
$$\doteq -619$$



So what does the dot product represent?

-“angular relationship” -somewhat similar to multiplication -properties very useful



Prove that two non-zero vectors $\vec{u}$ and $\vec{v}$ are perpendicular ***if and only if*** $\vec{u} \cdot \vec{v} = 0$.

if $\vec{u} \cdot \vec{v} = 0$

$|\vec{u}||\vec{v}|\cos\theta = 0$

$\cos\theta = 0$

$\theta = 90^\circ$

$\therefore \vec{u} \perp \vec{v}$

if $\vec{u} \perp \vec{v}$

$\theta = 90^\circ$

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos 90^\circ$

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\, 0$

$\vec{u} \cdot \vec{v} = 0$

What do you get if take the dot product of a vector with itself?

$\vec{x} \cdot \vec{x} = |\vec{x}||\vec{x}| \cos 0$

$= |\vec{x}|^2$

The Dot Product for Algebraic Vectors

Let $\vec{u} = (a_1, b_1, c_1)$ and $\vec{v} = (a_2, b_2, c_2)$.

$\vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2 + c_1 c_2$

Example: Find the dot product between the vectors $\vec{a} = (1, -3, 5)$ and $\vec{b} = (-2, 0, -4)$.

$\vec{a} \cdot \vec{b} = 1(-2) + (-3)(0) + 5(-4)$

$\vec{a} \cdot \vec{b} = -22$

Find the angle between these 2 vectors.

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$\cos\theta = \dfrac{-22}{\sqrt{1^2 + (-3)^2 + 5^2}\,\sqrt{(-2)^2 + (-4)^2}}$

$\cos\theta = \dfrac{-22}{\sqrt{35}\,\sqrt{20}}$

$\theta \doteq 146°$

Show that vectors $\vec{a} = (15, 0, -3)$ and $\vec{b} = (-1, 5, 5)$ are perpendicular.

$\vec{a} \cdot \vec{b} = 15(-1) + 0 - 3(5)$

$= -30$

they are not $\perp$.

Text page 177 #1 -2, 4abc, 5 – 9, 11, 13, 24