

## The Dot Product

Today we will define a new operation between 2 vectors – called the dot product. Unlike previous operations with vectors this operation is used for vectors and does not have a similar operation with scalars.

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### The Dot Product

The *dot product* of two vectors  $\vec{u}$  and  $\vec{v}$  is given by:

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos \theta$$

where  $\theta$  is the angle between the 2 vectors (tail-to-tail).

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The dot product is sometimes called the *scalar product* or *an inner product*.

**Example** Find  $\vec{u} \cdot \vec{v}$  if:

a)  $|\vec{u}| = 7$ ,  $|\vec{v}| = 15$  and  $\theta = 60^\circ$ .

b)  $|\vec{u}| = 13$ ,  $|\vec{v}| = 55$  and  $\theta = 150^\circ$ .

So what does the dot product represent?

-“angular relationship”

-somewhat similar to multiplication

-properties very useful

Prove that two non-zero vectors  $\vec{u}$  and  $\vec{v}$  are perpendicular **if and only if**  $\vec{u} \cdot \vec{v} = 0$ .

What do you get if take the dot product of a vector with itself?

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### The Dot Product for Algebraic Vectors

Let  $\vec{u} = (a_1, b_1, c_1)$  and  $\vec{v} = (a_2, b_2, c_2)$ .

$\vec{u} \cdot \vec{v} =$

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**Example:** Find the dot product between the vectors  $\vec{a} = (1, -3, 5)$  and  $\vec{b} = (-2, 0, -4)$ .

Find the angle between these 2 vectors.

Show that vectors  $\vec{a} = (15, 0, -3)$  and  $\vec{b} = (-1, 5, 5)$  are perpendicular.