Unit #1 Test: Vectors MCV4U

Name: Solutions

Marking Summary:

Knowledge/Understanding: questions #1, 4

Application: questions #3, 6

Thinking/Inquiry and Problem Solving: #2

Communication: all

Total Marks: _____

40

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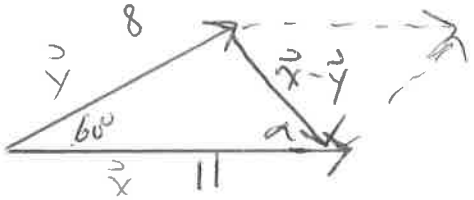
1. Let $\vec{x}$ and $\vec{y}$ be vectors with $|\vec{x}| = 11$, $|\vec{y}| = 8$. The angle between $\vec{x}$ and $\vec{y}$ is 60°. Find $|2\vec{x} - \vec{y}|$ and the angle $2\vec{x} - \vec{y}$ makes with vector $\vec{x}$. (round to nearest degree) **[5 marks]**



$|\vec{x}-\vec{y}|^2 = 8^2+11^2-2(1)(11)\cos 60^\circ$

$|\vec{x}-\vec{y}| = \sqrt{97}$

$\doteq 9.85$

$\frac{\sin\alpha}{8} = \frac{\sin 60^\circ}{\sqrt{97}}$

$\sin\alpha = \frac{8\sin 60^\circ}{\sqrt{97}}$

$\alpha \doteq 44.7^\circ$

so $\vec{x}-\vec{y}$ makes angle of 44.7° with $\vec{x}$ (45°)

2. In hexagon ABCDEF, opposite sides are parallel and equal in length. In addition, **FC = 2AB**.
Let $\overrightarrow{AB} = \vec{x}$ and $\overrightarrow{FA} = \vec{y}$.

Express each of the following in terms of $\vec{x}$ and $\vec{y}$. (You just need to show a correct answer, no justification required). The figure is shown three times just in case you make a mess! **[6 marks]**

a) $\overrightarrow{CF} = -2\vec{x}$

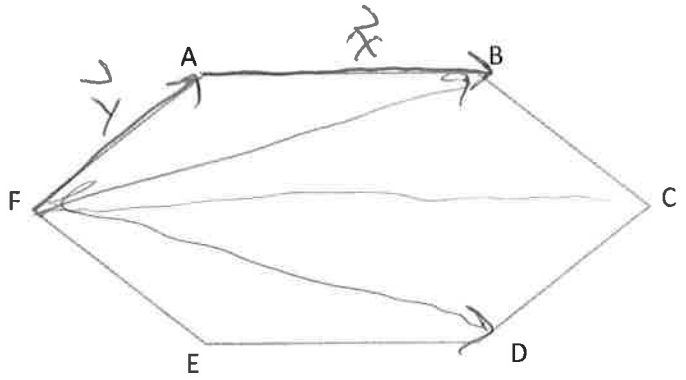
b) $\overrightarrow{FB} = \vec{x} + \vec{y}$

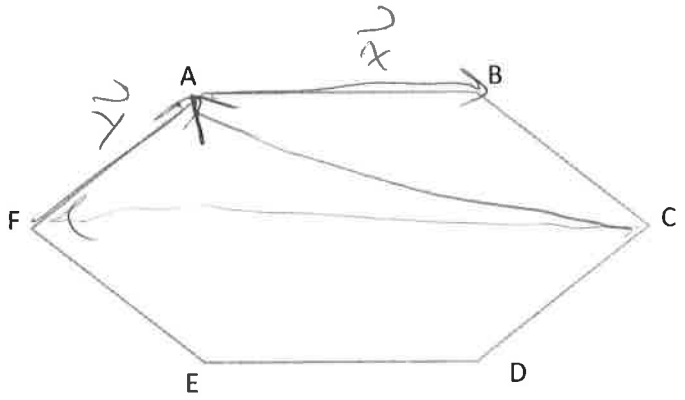
c) $\overrightarrow{FD} = \vec{FC} + \vec{CD}$
$= 2\vec{x} - \vec{y}$

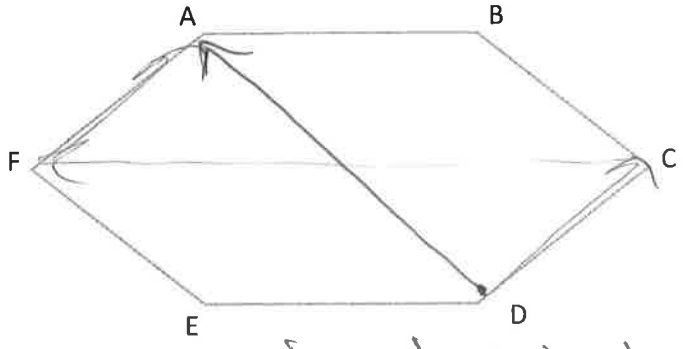
d) $\overrightarrow{DF} = -(2\vec{x} - \vec{y})$
$= -2\vec{x} + \vec{y}$

e) $\overrightarrow{CA}$
$= -2\vec{x} + \vec{y}$

f) $\overrightarrow{DA}$
$= 2\vec{y} - 2\vec{x}$

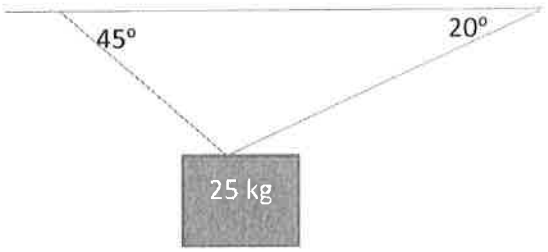






$\vec{DA} = \vec{y} - 2\vec{x} + \vec{y}$

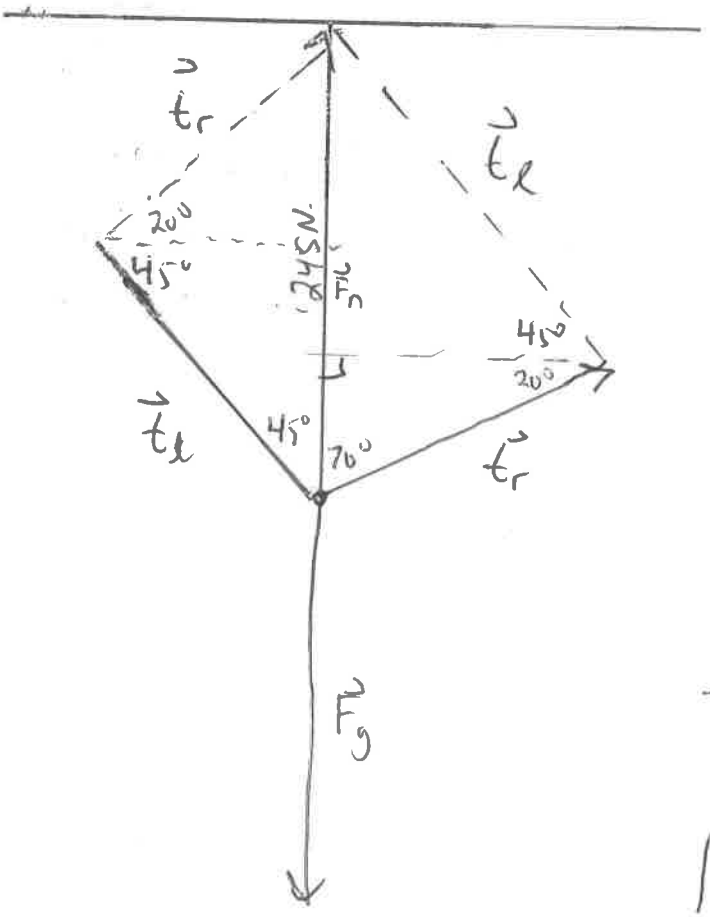
3. A 25kg mass is suspended from a ceiling by two cords. The cords make angles of 20° and 45° with the ceiling as shown. Determine the tension in each cord.
(HINT: Force due to gravity = 9.8N/kg) **[6 marks]**



$25^{kg} \times 9.8\ ^{N}/_{kg}$

$|\vec{F}_g| = 245N.$ and $|\vec{F}_n| = 245$

Force Diagram



$$\frac{|\vec{t}_r|}{\sin 45^\circ} = \frac{245}{\sin 65^\circ}$$

$$|\vec{t}_r| = \frac{245 \sin 45^\circ}{\sin 65^\circ}$$

$$|\vec{t}_r| = 191N$$

$$\frac{|\vec{t}_\ell|}{\sin 70^\circ} = \frac{245}{\sin 65^\circ}$$

$$|\vec{t}_\ell| = \frac{245 \sin 70^\circ}{\sin 65^\circ}$$

$$|\vec{t}_\ell| = 254N$$

∴ tension is 254N and 191N.

4. Let $\vec{a} = (3,-1,1)$, $\vec{b} = (-1,0,1)$ and $\vec{c} = (-6,3,-3)$. **[12 marks]**

a) Find $\vec{a} + 2\vec{b} - \vec{c}$

b) Find $|\vec{a}|$.

c) Find the angle that vector $\vec{a}$ makes with the positive z-axis. (nearest degree)

d) Find a unit vector in the direction of $\vec{b}$.

e) Are any of these vectors collinear? Explain.

f) Find a vector that has a magnitude of 10 and the opposite direction as vector $\vec{b}$.

a) $(3,-1,1) + 2(-1,0,1) - (-6,3,-3)$

$= (3,-1,1) + (-2,0,2) - (-6,3,-3)$

$= (7,-4,6)$ ✓✓

b) $|\vec{a}| = \sqrt{3^2+(-1)^2+1^2}$

$|\vec{a}| = \sqrt{11}$ ✓

c) $\cos\gamma = \frac{1}{\sqrt{11}}$ ✓

$\gamma \doteq 72^\circ$

d) $\hat{b} = \frac{\vec{b}}{|\vec{b}|}$

$\hat{b} = \frac{(-1,0,1)}{\sqrt{2}}$ ✓

$\hat{b} = \left(\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)$

e) No. There is no k such that $\vec{a} = k\vec{b}$ or $\vec{a} = k\vec{c}$ or $\vec{b} = k\vec{c}$.

f) $-10\hat{b} = -10\left(\frac{-\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)$

$= (5\sqrt{2}, 0, -5\sqrt{2})$ ✓✓

5. A, B and C are points such that A(1,1,-2), B(4, -8, 0) and C(-2, 10, -4).

a) Find the length of the line segment joining points A and B. **[5 marks]**

b) Are points A, B and C collinear? Justify your answer.

a) $\vec{AB} = (3, -9, 2)$

$|\vec{AB}| = \sqrt{3^2 + (-9)^2 + 2^2}$

$= \sqrt{94}$

b) $\vec{AB} = (3, -9, 2)$ $\vec{BC} = (-6, 18, -4)$

$\vec{BC} = -2\vec{AB}$ or $\vec{AB} = -\frac{1}{2}\vec{BC}$

∴ yes, points A, B and C are collinear.

6. A small airplane has an air speed of 244 km/h. The pilot wishes to fly to a destination that is 480 km due West from the plane's present location. There is a 44 km/h wind from the south (travelling North).

[6 marks]

a) In what direction should the pilot fly in order to reach the destination?

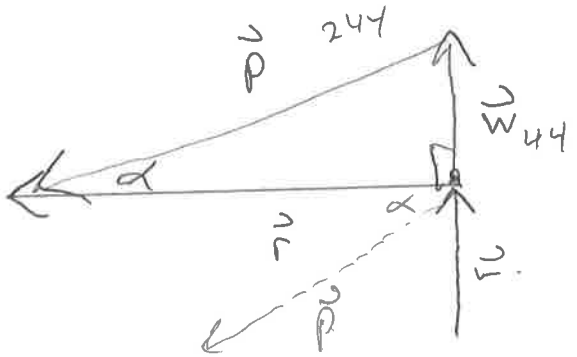
b) What is the plane's resultant speed relative to the ground?

Let air velocity $= \vec{p}$

wind velocity $= \vec{w}$

$\vec{r} = \vec{w} + \vec{p}$

$\vec{r}$ = resultant (ground velocity)



$244^2 = 44^2 + |\vec{r}|^2$

$|\vec{r}|^2 = 244^2 - 44^2$

$|\vec{r}| = 240 \text{ km/h}$

b) 240 km/h.

$\sin\alpha = \frac{44}{244}$

$\alpha \doteq 10.4°$

∞ plane should travel West, 10.4° South or on a bearing of 259.4°.