MCV4U Lesson 2

Properties of the Dot Product

Similar to vector addition and scalar multiplication, there are many properties that hold true with the dot product.

Vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $k \in \mathbb{R}$

Commutative $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

Scalar Multiplication $k(\vec{a} \cdot \vec{b}) = (k\vec{a}) \cdot \vec{b} = \vec{a} \cdot (k\vec{b})$

Distributive $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

Associative $(\vec{a} \cdot \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \cdot \vec{c})$ ← meaningless

***Recall that: $\vec{a} \cdot \vec{a} = |\vec{a}|^2$ (from yesterday) ***

The distributive property allows us to "expand and simplify" expressions with the dot product.

Examples

$(2\vec{a} + \vec{b}) \cdot (3\vec{a} - \vec{b})$

$= 6\vec{a} \cdot \vec{a} - 2\vec{a} \cdot \vec{b} + 3\vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{b}$

$= 6|\vec{a}|^2 + \vec{a} \cdot \vec{b} - |\vec{b}|^2$

$(2\hat{i} - \hat{j}) \cdot (3\hat{i} + 2\hat{j})$

$= 6|\hat{i}|^2 + \hat{i} \cdot \hat{j} - 2|\hat{j}|^2$

$= 6(1) + \hat{i} \cdot \hat{j} - 2(1)$

$= 6 + 0 - 2 = 4$

Expand and evaluate for $|\vec{a}| = 3$, $|\vec{b}| = 2$ and $\theta = 120°$.

$(2\vec{a} + 3\vec{b}) \cdot (4\vec{a} + 5\vec{b})$

$= 8|\vec{a}|^2 + 22\vec{a} \cdot \vec{b} + 15|\vec{b}|^2$

$= 8|\vec{a}|^2 + 22|\vec{a}||\vec{b}|\cos\theta + 15|\vec{b}|^2$

$= 8(3)^2 + 22(3)(2)\cos 120° + 15(2)^2$

$= 72 + 132\left(-\frac{1}{2}\right) + 60$

$= 66$

Two vectors $8\vec{u} - \vec{v}$ and $4\vec{u} + 3\vec{v}$ are perpendicular and the magnitude of $\vec{v}$ is twice the magnitude of $\vec{u}$. Find the angle between vectors $\vec{v}$ and $\vec{u}$.

$(8\vec{u}-\vec{v})\cdot(4\vec{u}+3\vec{v})=0$

$32|\vec{u}|^2+20\vec{u}\cdot\vec{v}-3|\vec{v}|^2=0$

$32|\vec{u}|^2+20|\vec{u}||\vec{v}|\cos\theta-3|\vec{v}|^2=0$ $\quad |\vec{v}|=2|\vec{u}|$

$32|\vec{u}|^2+20(\vec{u})\,2|\vec{u}|\cos\theta-3(2|\vec{u}|)^2=0$

$20|\vec{u}|^2+40|\vec{u}|^2\cos\theta=0$

$20|\vec{u}|^2(1+2\cos\theta)=0$

$20|\vec{u}|^2=0$ $\quad 1+2\cos\theta=0$

$|\vec{u}|=0$ $\quad \cos\theta=-\frac{1}{2}$

trivial solution $\quad \theta=120^\circ$

Text page 179 #14, 15, 16, 17, 18, 19, 22, 23 (can be completed with exact values)