The Cross Product

The cross product is an operation that given two vectors $\vec{a}$ and $\vec{b}$ will return a vector that is perpendicular to both $\vec{a}$ and $\vec{b}$. Hence, the cross product is only defined in 3-space. WHY?

The cross product or vector product of $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ is given by:

$\vec{a} \times \vec{b} =$

Example: Let $\vec{u} = (1,2,-4)$ and $\vec{v} = (2,-3,1)$. Find $\vec{u} \times \vec{v}$. Verify this new vector is perpendicular to $\vec{u}$ and $\vec{v}$.

If $\vec{a}$ and $\vec{b}$ are non-collinear vectors in R^3 then every vector that is perpendicular to both $\vec{a}$ and $\vec{b}$ will have the form $k(\vec{a} \times \vec{b})$ for some $k \epsilon R$.

We won't prove the above statement. But we can demonstrate that the cross product does in fact produce a vector perpendicular to the two given vectors. Let $\vec{v} = \vec{a} \times \vec{b}$. Verify that $\vec{v}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

Example: Find 3 vectors perpendicular to both $\vec{x} = (8,4,0)$ and $\vec{y} = (-2,1,6)$

How can we express the cross-product geometrically?

Direction of cross product is given by a "right-handed system". Wrap fingers from a to b, thumb points in direction of cross product (see page 183-184)

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