MCV4U **Assignment #1: Vectors** due date: Friday Feb. 28th

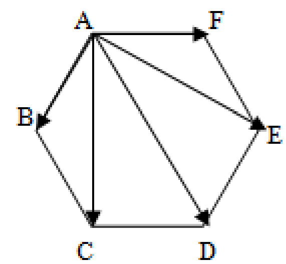
-each problem is worth 10 marks.

-only selected problems will be submitted on the due date.

-be prepared to submit the selected questions in numerical order.

-this is an individual assignment

1. a) Consider a regular hexagon *ABCDEF* as shown. Prove that $AB + AC + AD + AE + AF = 3AD$



b) In triangle ABC, a median is drawn from vertex A to the midpoint of BC, which is labelled *D*. If $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$, prove that $\overrightarrow{AD} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c}$.

2. Let $\vec{x}$ and $\vec{y}$ be vectors in R^3. Prove that $\vec{x} \cdot \vec{y} = \frac{1}{2}(|\vec{x} + \vec{y}|^2 - |\vec{x}|^2 - |\vec{y}|^2)$:
   a) Using a geometric approach.
   b) Using an algebraic approach (i.e. using components)

3. a) If $\vec{a} = (2,3,7)$ and $\vec{b} = (-4, y, -14)$, for what values of *y* are $\vec{a}$ and $\vec{b}$ perpendicular? For what values of *y* are they collinear?

   b) Find ***two unit vectors*** that are perpendicular to both vectors $\vec{x} = (2,0,-1)$ and $\vec{y} = (-3,-4,1)$. Are there any more that exist? (Explain)

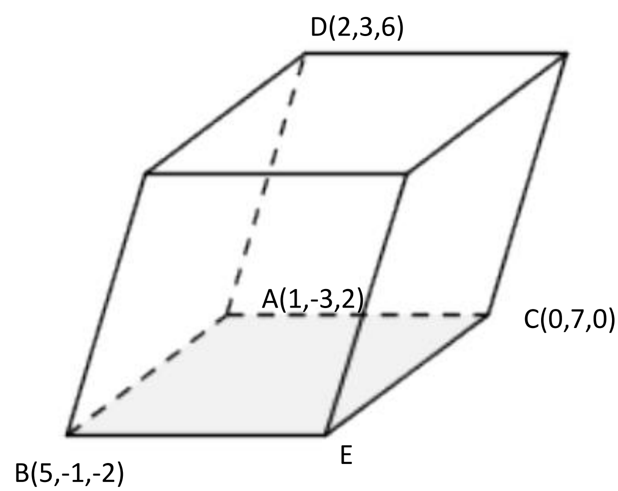
4. A river is 2km wide and flows at 6 km/h. Anna is driving a motorboat that has a speed of 20 km/h in still water. A marina lies directly across the river from her starting point. If Anna decides she wants to end up directly across the river at the marina, in what direction should she head? How long will it take her to get there?

5. a) Find the area of the triangle formed by the points A(1, 2, -1), B(5, 9, 0) and C(-6, 0, 3).

   b) Determine the angle between $\vec{a}$ and $\vec{b}$ given that $\hat{a} - 5\hat{b}$ and $\hat{a} - \hat{b}$ are perpendicular.

6. A parallelepiped is drawn with four of its vertices as shown below.

   a) Find the volume of the parallelepiped.
   b) Find the coordinates of point E.
   c) Find the acute angle of intersection between the face diagonal AE and BC. (nearest degree)



7. a) Prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2\vec{a} \times \vec{b}$.

   b) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-zero vectors such that $\vec{c} = \vec{a} \times \vec{b}$. If $s, t \epsilon R$ and $\vec{d} = s\vec{a} + t\vec{b}$, then determine the value of $\vec{c} \cdot \vec{d}$. Explain (in words).