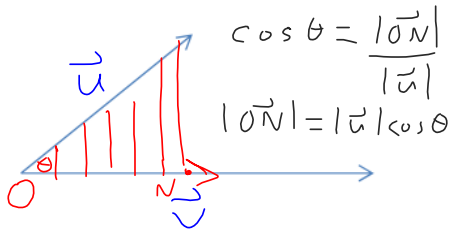
MCV4U

# Applications of Dot and Cross Products

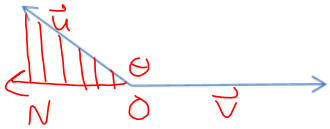
Mathematically, a projection is formed by drawing perpendicular lines from each of the points on one object onto a line or plane.

The projection of one vector onto another can be pictured as follows.




$\cos\theta = \frac{|\vec{ON}|}{|\vec{u}|}$

$|\vec{ON}| = |\vec{u}|\cos\theta$




Proj. $\vec{u}$ onto $\vec{v}$ $= \vec{ON}$

$|\vec{ON}| = |$Proj. of $\vec{u}$ onto $\vec{v}|$

$= \left||\vec{u}|\cos\theta\right| \frac{|\vec{v}|}{|\vec{v}|}$

$= \frac{||\vec{u}||\vec{v}|\cos\theta|}{|\vec{v}|}$

$|$Proj. of $\vec{u}$ onto $\vec{v}| = \frac{|\vec{u}\cdot\vec{v}|}{|\vec{v}|}$ (magnitude of projection)

$\vec{ON} = \frac{(\vec{u}\cdot\vec{v})}{|\vec{v}|}\hat{v} = \frac{(\vec{u}\cdot\vec{v})}{|\vec{v}|}\frac{\vec{v}}{|\vec{v}|}$

Proj of $\vec{u}$ onto $\vec{v} = \frac{(\vec{u}\cdot\vec{v})\vec{v}}{|\vec{v}|^2}$ ← projection vector

**Example** Find the projection of vector $\vec{a} = (2,0,-4)$ onto vector $\vec{b} = (1,3,8)$.

Proj of $\vec{a}$ onto $\vec{b} = \frac{(\vec{a}\cdot\vec{b})\vec{b}}{|\vec{b}|^2}$

$= \frac{-30(1,3,8)}{1^2+3^2+8^2}$

$= \frac{(-30,-90,-240)}{74} = \left(\frac{-30}{74}, \frac{-90}{74}, \frac{-240}{74}\right)$

**Example** Find the projection of the vector $\vec{v} = (4,-8)$ on the basis vectors $\hat{\imath}$ and $\hat{\jmath}$.

Proj $\vec{v}$ onto $\hat{i} = (4,0)$

Proj $\vec{v}$ onto $\hat{j} = (0,-8)$

$\hat{i} = (1,0)$

$\hat{j} = (0,1)$

## WORK

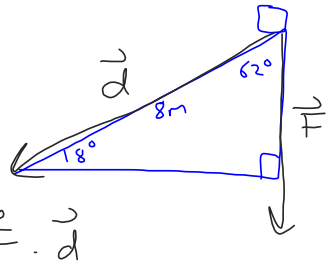
In physics, work is done whenever a force acting on an object causes displacement of that object. Work is defined as the following:

$W = \vec{F} \cdot \vec{d}$ where $\vec{F}$ is the force acting on an object and $\vec{d}$ is the displacement (movement) of that object.

The unit for work is the Joule (J). Note that $J = N \times m = \frac{kg \times m^2}{s^2}$ $W = |\vec{F}||\vec{d}|\cos\theta$

**Example**

A 25-kg box is located 8 m up a ramp inclined at 18° to the horizontal. Determine the work done by the force of gravity as the box slides to the bottom of the ramp.




$|\vec{d}| = 8m$

$|\vec{F}| = 25kg \times 9.8 N/kg$

$= 245N$

$W = \vec{F} \cdot \vec{d}$

$= 245N(8m)\cos 62°$

$= 920 J$

**Torque**

A force that causes an object to turn is called torque. (Consider using a wrench).

Torque can be calculated using the cross product:

$\vec{T} = \vec{r} \times \vec{F}$

$|\vec{T}| = |\vec{r}||\vec{F}| \sin\theta$

where: $\vec{F}$ is the applied force, $\vec{r}$ is the vector determined from the lever arm acting from axis of rotation

$\theta$ is the angle between $\vec{r}$ *and* $\vec{F}$.

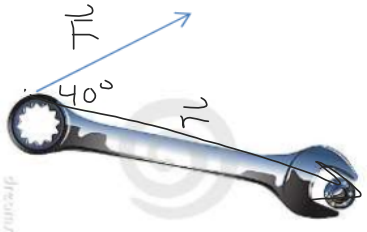
**Example**: A force of 50 N acting at an angle of $40^0$ at the end of a 19 cm wrench as shown in the diagram below. Calculate the torque on the bolt.

$\vec{T} = \vec{r} \times \vec{F}$

$|\vec{T}| = |\vec{r}||\vec{F}| \sin\theta$

$= (.19m)(50N)\sin 40°$

$|\vec{T}| = 6.1 \ N \cdot m$ "into the page"



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