MCV4U

Properties and Applications of the Cross Product

Properties of the Cross Product

Let $\vec{a}, \vec{b}$ and $\vec{c}$ be vectors in three-dimensional space and let $k \in R$.

$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$ anti-commutative

$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ distributive

$k(\vec{a} \times \vec{b}) = k\vec{a} \times \vec{b} = \vec{a} \times k\vec{b}$

$\vec{a} \times \vec{a} = \vec{0}$

Two non-zero **vectors $\vec{a}$ and $\vec{b}$ are collinear if and only if $\vec{a} \times \vec{b} = \vec{0}$**

if $\vec{a} \times \vec{b} = \vec{0}$

$|\vec{a}||\vec{b}|\sin\theta = 0$

$\sin\theta = 0$

$\theta = 0°$ or $\theta = 180°$ and $\vec{a}, \vec{b}$ are collinear

if $\vec{a}, \vec{b}$ are collinear

$\theta = 0°$ or $180°$ and $\sin\theta = 0$

$|\vec{a}||\vec{b}|\sin\theta = 0$

$|\vec{a} \times \vec{b}| = 0$ and $\vec{a} \times \vec{b} = \vec{0}$

Triple Scalar Product

The triple scalar product between vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is defined as $(\vec{a} \times \vec{b}) \cdot \vec{c}$

If $(\vec{a} \times \vec{b}) \cdot \vec{c} = 0$ what can you conclude? $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

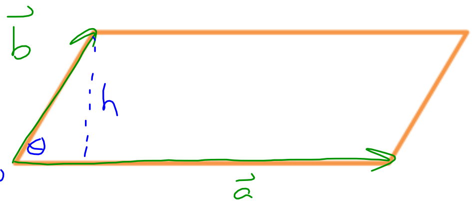
If $(\vec{a} \times \vec{b}) \cdot \vec{c} \neq 0$ what can you conclude? non-coplanar

Area of a Parallelogram

$A = |\vec{a}| \times h$

$\sin\theta = \frac{h}{|\vec{b}|}$

$h = |\vec{b}|\sin\theta$

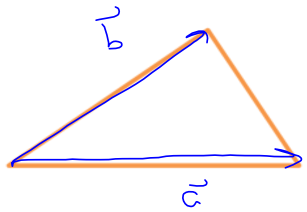


$A = |\vec{a}||\vec{b}|\sin\theta$

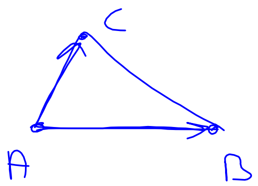
$A = |\vec{a} \times \vec{b}|$

Area of a Triangle

$A = \frac{|\vec{a} \times \vec{b}|}{2}$



Example Find the area of the triangle having vertices A(1,2,3), B(-3,2,1) and C(2,3,-1)



$\vec{AC} = (1, 1, -4)$

$\vec{AB} = (-4, 0, -2)$

$$A = \frac{|\vec{AC} \times \vec{AB}|}{2}$$

$$A = \frac{|(-2, 18, 4)|}{2}$$

$$A = \frac{\sqrt{(-2)^2 + 18^2 + 4^2}}{2}$$

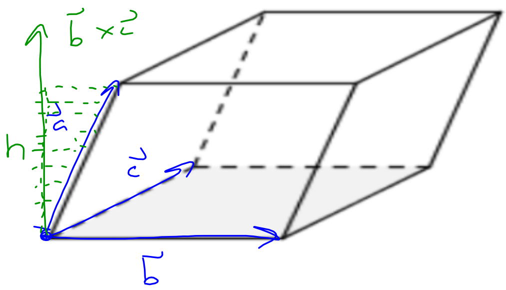
Volume of a Parallelepiped

Volume = Area of base x height

$= |\vec{b} \times \vec{c}| \times \text{height}$

$= |\vec{b} \times \vec{c}| \times \dfrac{|\vec{a} \cdot (\vec{b} \times \vec{c})|}{|\vec{b} \times \vec{c}|}$

$= |\vec{a} \cdot (\vec{b} \times \vec{c})|$



height $= |\text{Proj. of } \vec{a} \text{ onto } \vec{b} \times \vec{c}|$

$= \dfrac{|\vec{a} \cdot (\vec{b} \times \vec{c})|}{|\vec{b} \times \vec{c}|}$

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