



Let θ be angle between diagonals.

Each diagonal is either $\vec{a} + \vec{b}$ or $\vec{a} - \vec{b}$.

Since it is a rectangle, diagonals are equal in length. $\therefore |\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a} + \vec{b}| |\vec{a} - \vec{b}| \cos \theta$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a} + \vec{b}|^2 \cos \theta \quad \text{since } |\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$$

$$|\vec{a}|^2 - \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} - |\vec{b}|^2 = |\vec{a} + \vec{b}|^2 \cos \theta$$

$$|\vec{a}|^2 - |\vec{b}|^2 = |\vec{a} + \vec{b}|^2 \cos \theta$$

$$\cos \theta = \frac{|\vec{a}|^2 - |\vec{b}|^2}{|\vec{a} + \vec{b}|^2}$$

but using pythagorean theorem:

$$\therefore \cos \theta = \frac{|\vec{a}|^2 - |\vec{b}|^2}{|\vec{a}|^2 + |\vec{b}|^2}$$

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$$

whether $|\vec{a}| \geq |\vec{b}|$ or $|\vec{a}| < |\vec{b}|$ changes whether you get obtuse or acute angle of intersection.