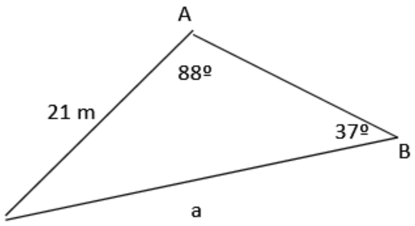
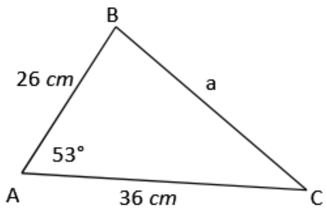
The Cosine Law

Warm-up

Solve for the indicated side in each triangle below.





$$\frac{a}{\sin 88} = \frac{21}{\sin 37}$$

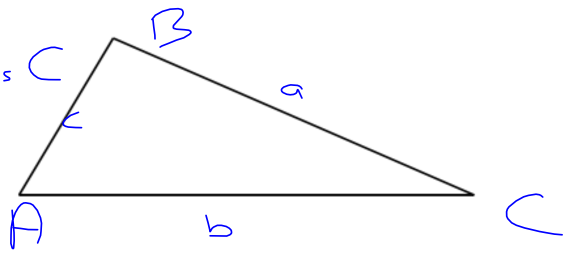
$$a = \frac{21 \sin 88°}{\sin 37°}$$

$$a \doteq 34.9\text{m}$$

We will now introduce the cosine law.

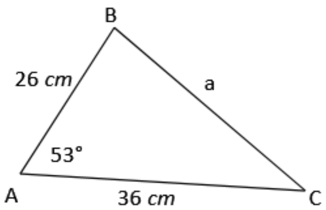
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\begin{cases} a^2 = b^2 + c^2 - 2bc\cos A \\ b^2 = a^2 + c^2 - 2ac\cos B \end{cases}$$



Use the Cosine Law to solve for the indicated sides in the triangles below.

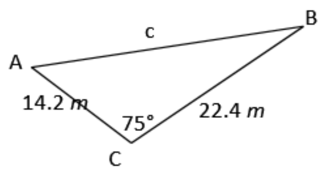
a)



$$a^2 = 26^2 + 36^2 - 2(26)(36)\cos 53°$$

$$a \doteq 29 \text{ cm}$$

b)



$$c^2 = a^2 + b^2 - 2ab\cos C$$

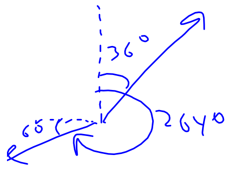
$$c^2 = 14.2^2 + 22.4^2 - 2(14)(22.4)\cos 75°$$

$$c \doteq 23.2\text{ m}$$

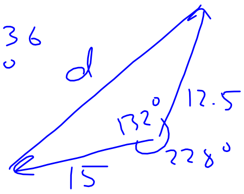
MCF3M — Unit 4, Lesson 5

Suppose two hikers leave from the same spot at the same time. The first hiker heads on a bearing of 36°, at a speed of 5 km/h. The second hiker leaves on a bearing of 264° at a speed of 6 km/h.

Find the distance between the two hikers after 2.5 hours.



$264 - 36 = 228°$



$360 - 228 = 132°$

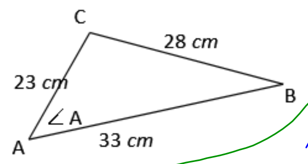
$6\ km/h \times 2.5\ h = 15\ km$

$5\ km/h \times 2.5\ h = 12.5\ km$

$d^2 = 15^2 + 12.5^2 - 2(15)(12.5)\cos 132°$

$d \doteq 25.1\ km$

We can also use the cosine law to solve for an angle when we have all 3 sides.



$a^2 = b^2 + c^2 - 2bc\cos A$

$28^2 = 23^2 + 33^2 - 2(23)(33)\cos A$

$28^2 - 23^2 - 33^2 = -1518\cos A$

$-834 = \frac{-1518\cos A}{-1518}$

$\frac{834}{1518} = \cos A$

$A = \cos^{-1}\left(\frac{834}{1518}\right)$

$A \doteq 57°$

$a^2 - b^2 - c^2 = -2bc\cos A$

$\frac{a^2 - b^2 - c^2}{-2bc} = \cos A$

$\cos A = \frac{-a^2 + b^2 + c^2}{2bc}$

$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$

or $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

Text page 214 #1, 2ab, 3ac, 4a, 7, 8